Wesleyan University

Department of Physics
Wave Transport in Complex Systems Lab

In-situ Physical Adjoint Computing in Multiple-Scattering Electromagnetic Environments for Wave Control

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Acknowledgement: Prof. Steven Johnson, (Simons Presentation 2023)

Outline

- General Problem: Challenges in Wireless Communications
- Methodologies

1. Time-Reversal Mirror Protocol
2. Linear and Nonlinear Wavefront Shaping Protocols
3. Cavity Shaping Protocols

- In-Situ Adjoint Method

1. General Principles of Adjoint Method
2. Physical Implementation in Complex Networks

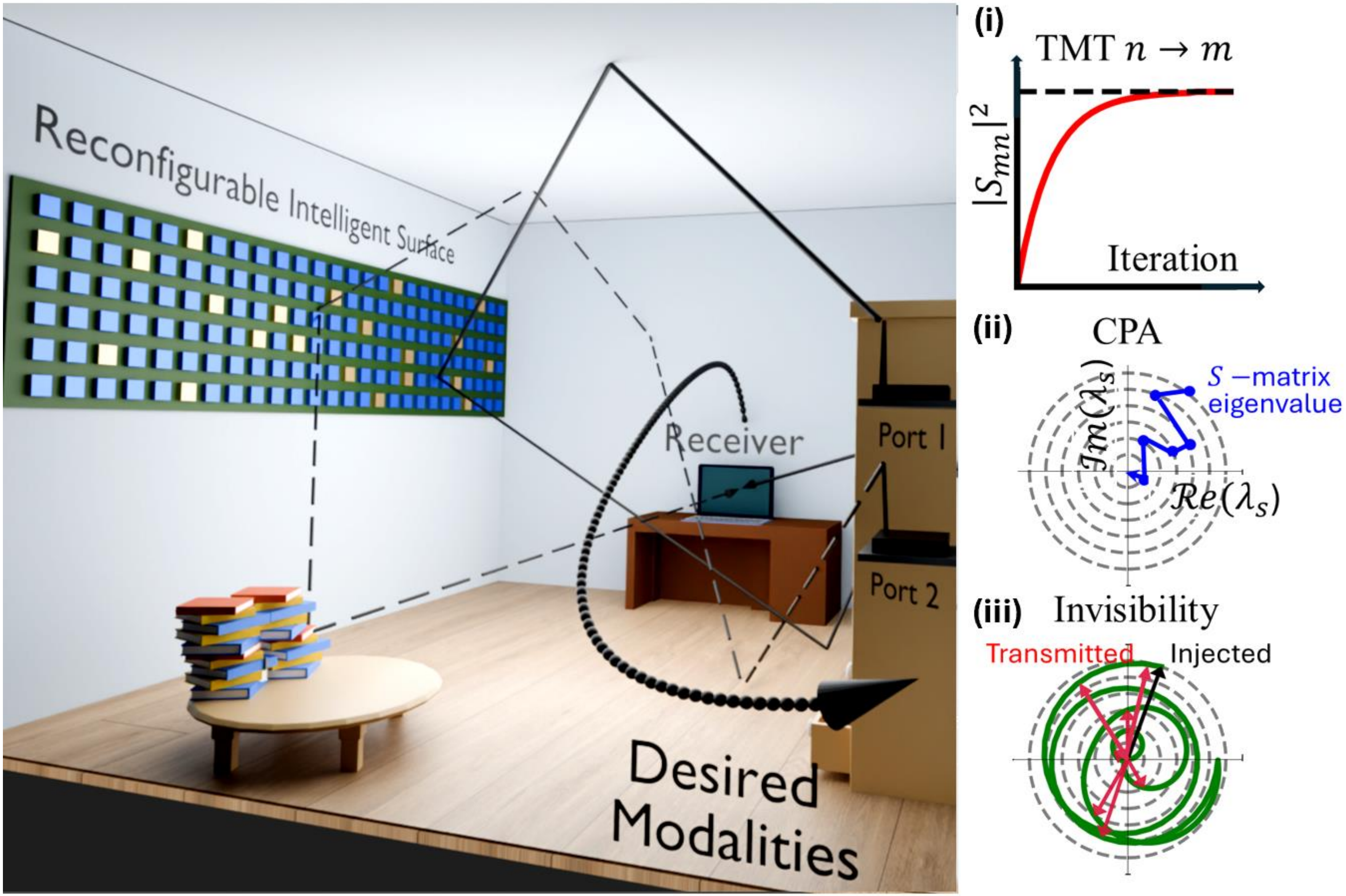
- In-Situ Modality Examples

1. Targeted Mode Transmission
2. Coherent Perfect Absorption
3. Invisibility

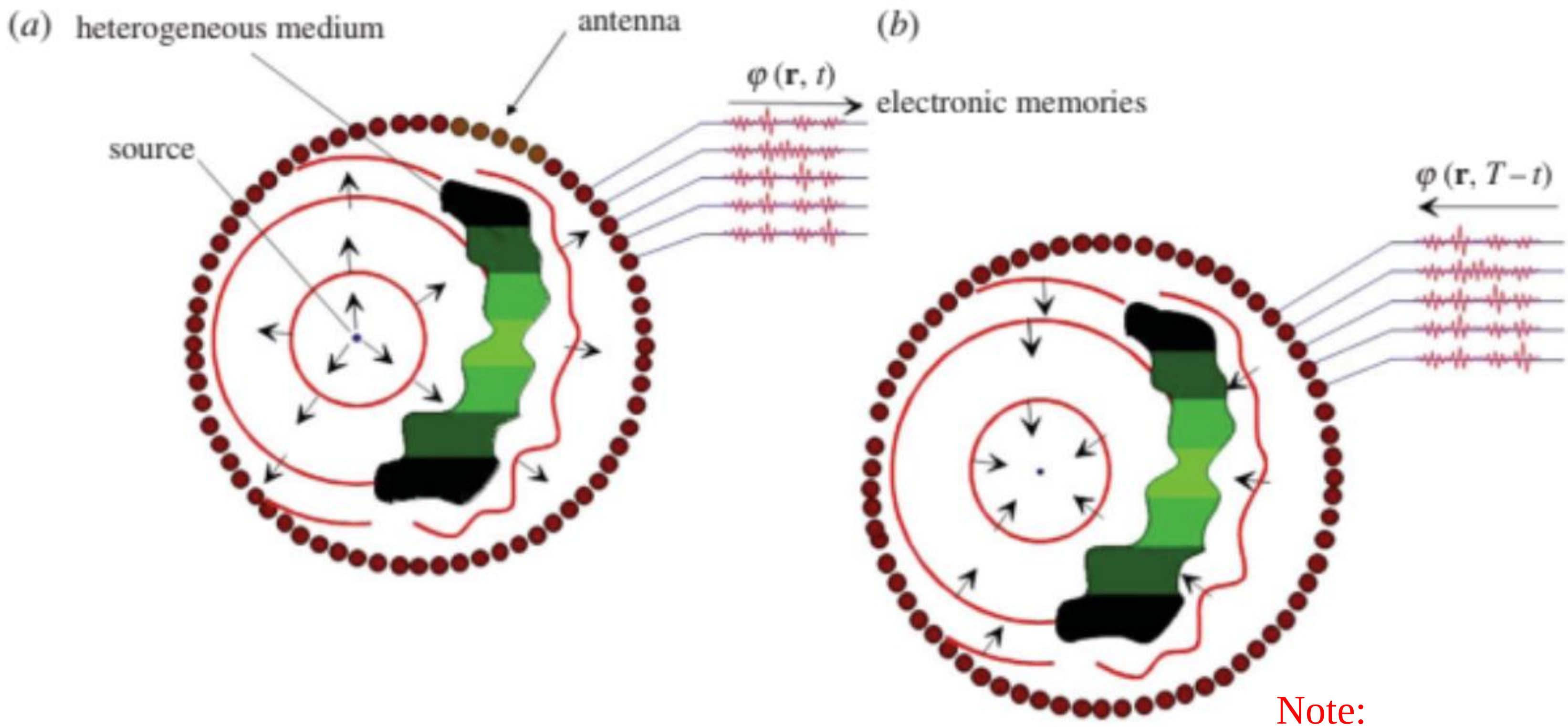
- In-Silico Generalization

- Outlook

Challenges in Wireless Communications



Time-Reversal Mirror Protocol



Time-Reversal Mirrors (TRM) in 4 steps:

(i)Target (source) emits a signal

(ii) Receivers (TRM) register the signal

(iii) TRM time-reverse the register signal

(iv) Send back the time-reverse signal

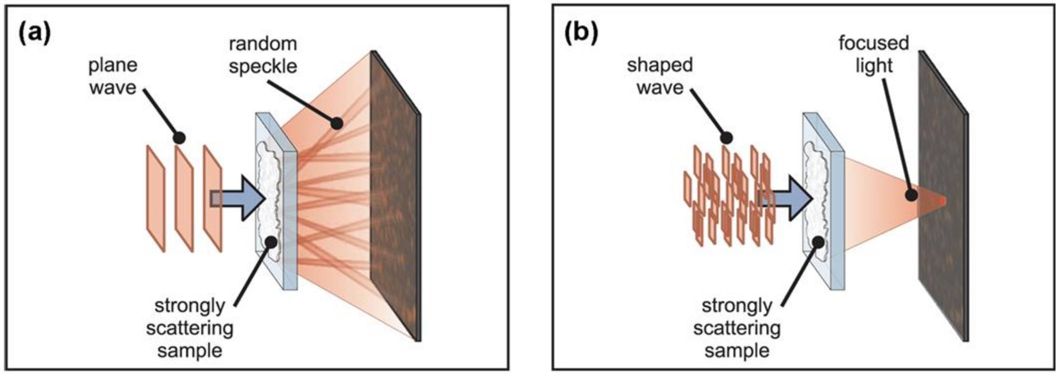
Note:

1) The ”environment” needs to be “static”

2) An exact TR process requires TR of source (from source to sink)

Fink, M. (2016). From Loschmidt daemons to time-reversed waves. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, *374*

Linear and Nonlinear Wavefront Shaping Protocols



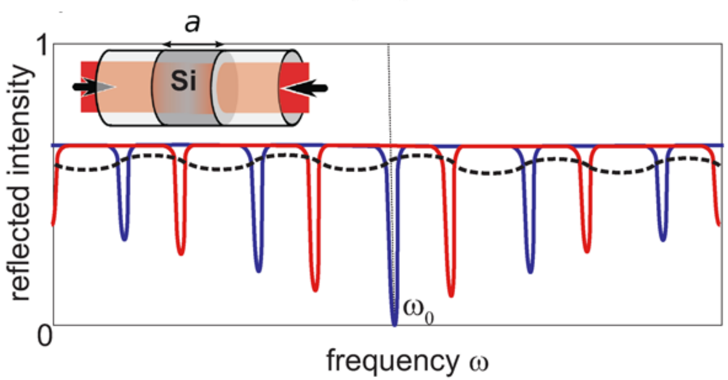
$$|\mathcal{O}\rangle = \hat{S}(k,\gamma)|\mathcal{J}\rangle$$

$$= \lambda(k,\gamma)|\mathcal{J}\rangle = 0$$

CPA Condition:

$$\lambda_{CPA}(k_{CPA} \in \mathcal{R}, \gamma_{CPA}) = 0$$

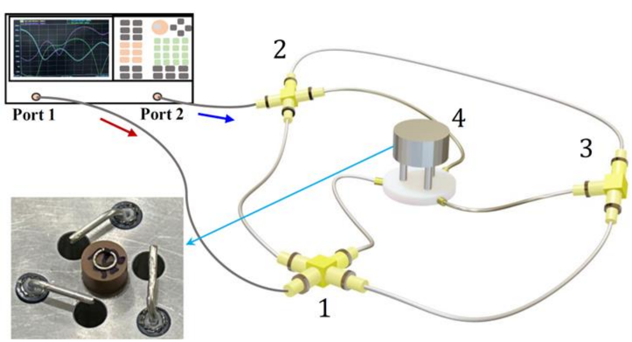
$$|\mathcal{J}\rangle = |\mathcal{J}_{CPA}\rangle$$

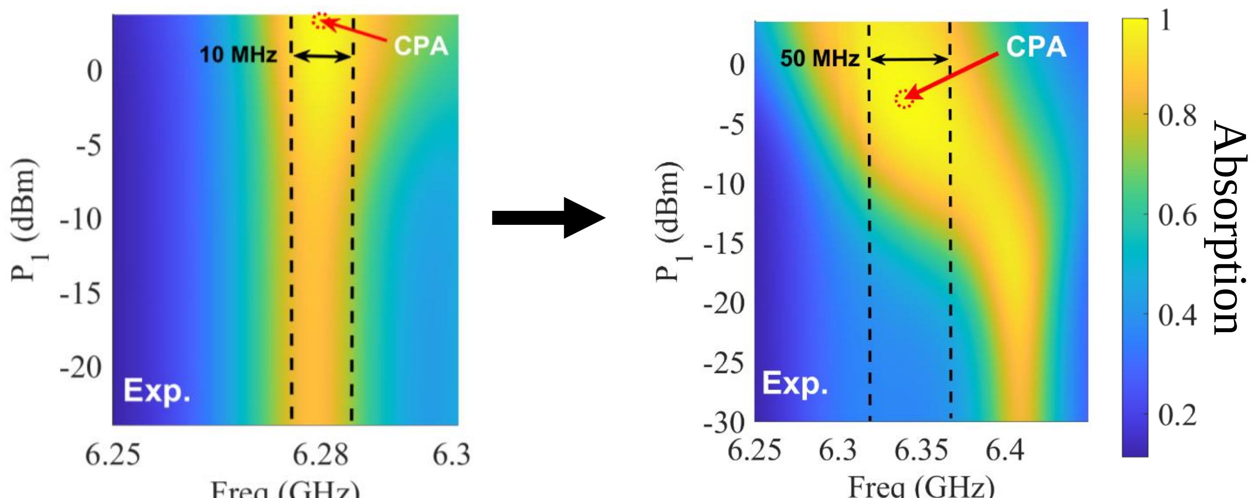


Vellekoop, I. M., & Mosk, A. P. (2007). Focusing coherent light through opaque strongly scattering media. *Optics letters*,

Chong, Y. D. Stone A.D. , et al. 2010, "Coherent perfect absorbers: time-reversed lasers."

Nonlinear Wavefront Shaping

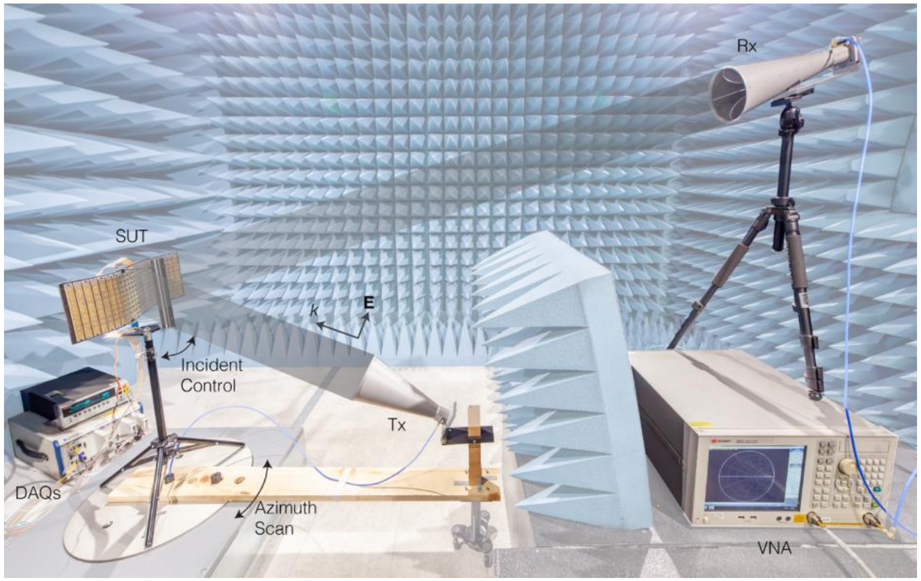


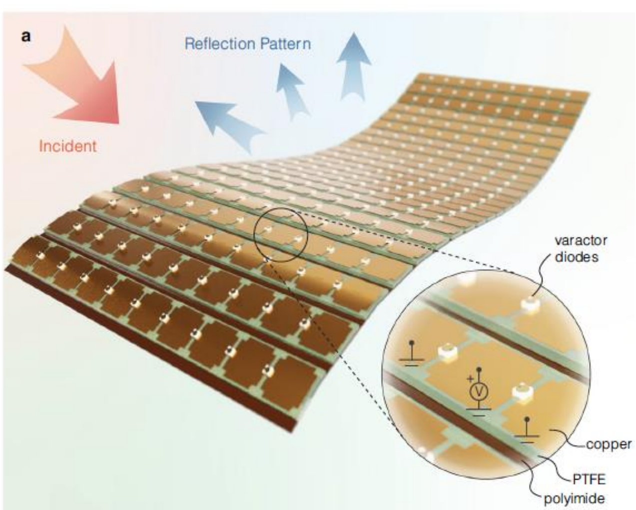


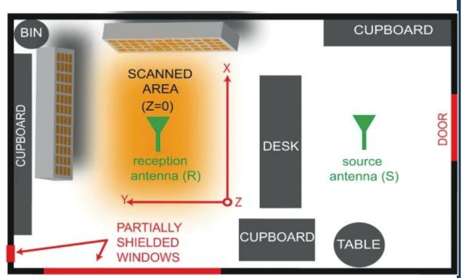
Wang, CZ, Guillamon, J. Tuxbury, W. et al. "Nonlinearity-induced scattering zero degeneracies for spectral management of coherent perfect absorption in complex systems." *Physical Review Applied* 22.6 (2024): 064093.

Cavity Shaping Protocols

Intelligent Wave Control in Complex Environment



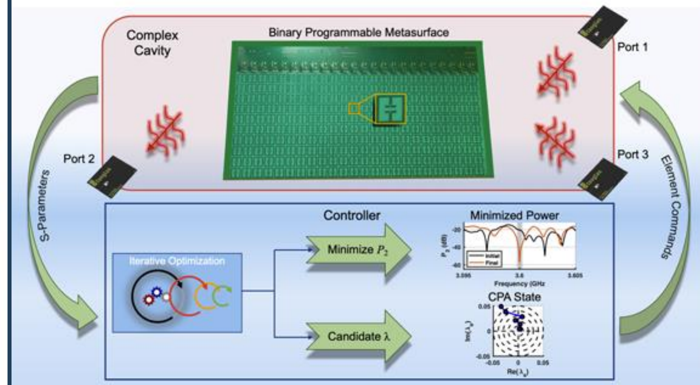




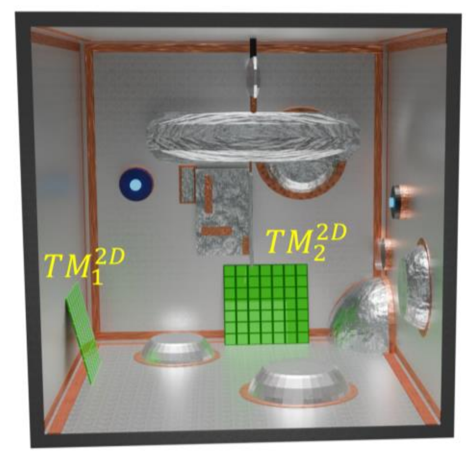
Kaina, N., Dupré, M., Lerosey, G., & Fink, M. (2014). Shaping complex microwave fields in reverberating media with binary tunable metasurfaces. *Scientific reports*, *4*(1), 6693.

Wen, Erda, Xiaozhen Yang, and Daniel F. Sievenpiper. "Real-data-driven real-time reconfigurable microwave reflective surface." *Nature Communications* 14.1 (2023): 7736.

Chaotic cavity/Meta-surface

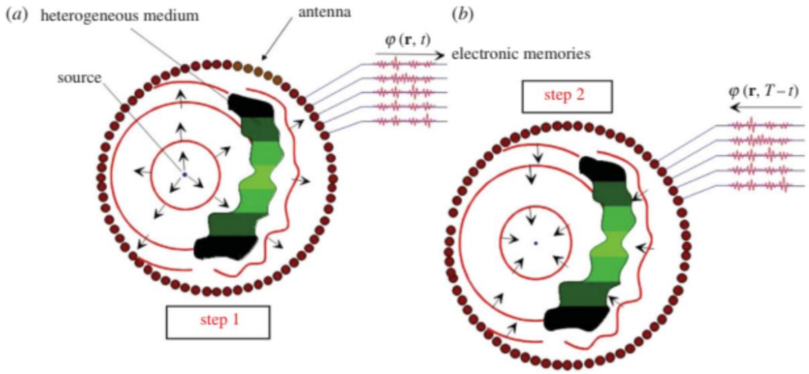


Frazier, B. W., Antonsen Jr, T. M., Anlage, S. M., & Ott, E. (2020). Wavefront shaping with a tunable metasurface: Creating cold spots and coherent perfect absorption at arbitrary frequencies. Physical Review Research, 2(4), 043422.

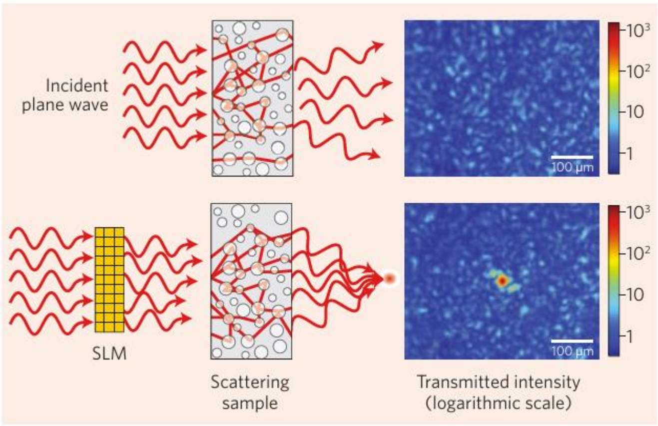


Erb, J., Shaibe, N., Calvo, R., Lathrop, D., Antonsen, T., Kottos, T., & Anlage, S. M. (2024). Novel Topology and Manipulation of Scattering Singularities in Complex non-Hermitian Systems. *arXiv preprint arXiv:2411.01069*.

Time Reversal



Wavefront Shaping



Cavity Shaping

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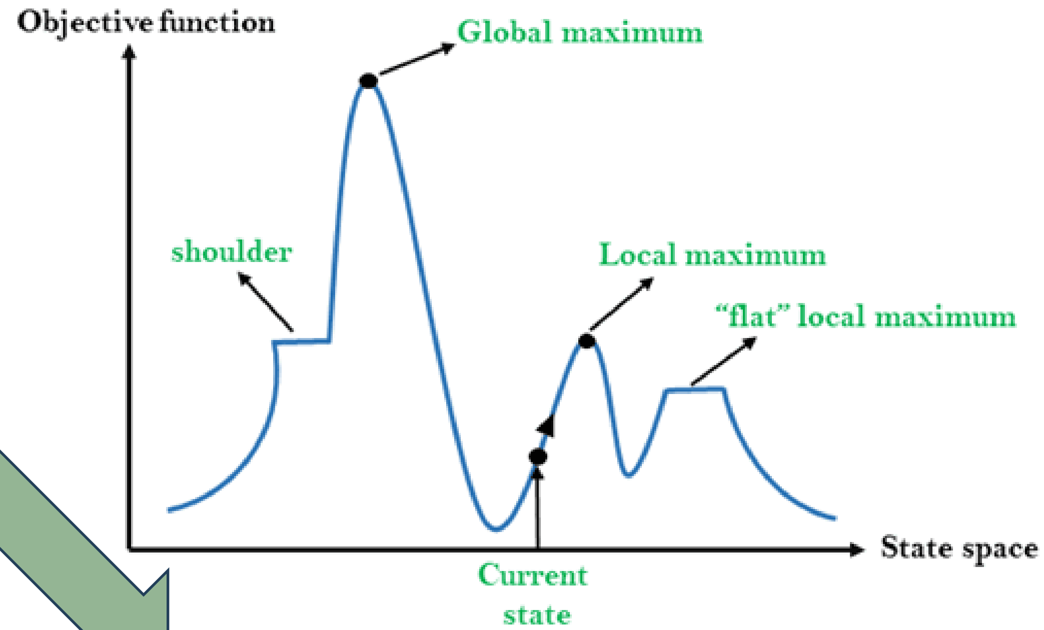
Adjoint-Based Gradient Optimization

Optimization



Gradient-Free:
Bayesian
Surrogate

Gradient-Based:
Finite Difference
Adjoint Method

We propose and demonstrate the use of an *in-situ* (Experimentally Driven) Adjoint Method

Principles of In-Situ Adjoint Optimization

$\mathbf{g}(\boldsymbol{p}, \boldsymbol{\Phi}, \boldsymbol{\Phi}^*)$: Objective

$\boldsymbol{p}$: Parameter Vector

$\boldsymbol{b}(\boldsymbol{p})$: Source

$\mathcal{M}(\boldsymbol{p})$: Wave System

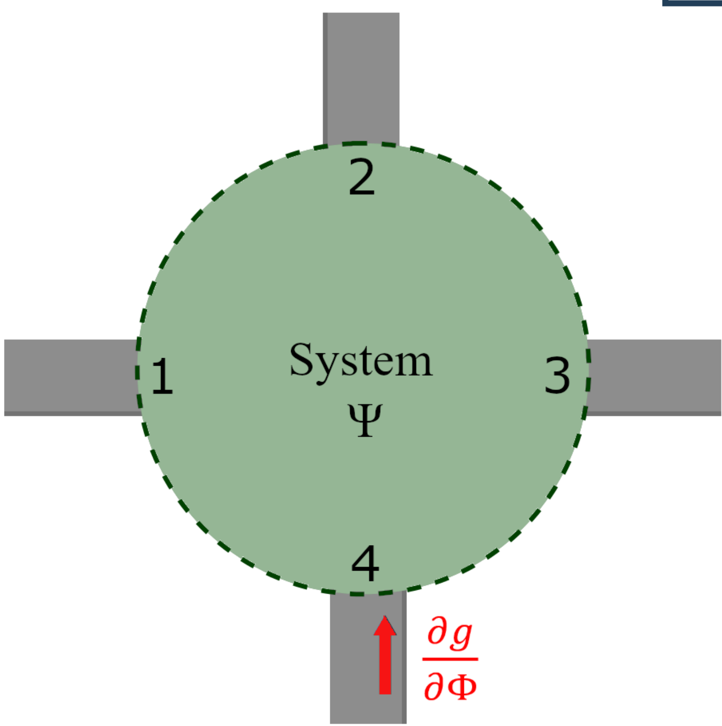
$\boldsymbol{\Phi}(\boldsymbol{p})$: Forward Field

$\boldsymbol{\lambda}(\boldsymbol{p})$: Adjoint Field

Forward Problem: $\mathcal{M}(\boldsymbol{p})\boldsymbol{\Phi} = \boldsymbol{b}(\boldsymbol{p})$

Adjoint Problem: $\mathcal{M}^{T}\boldsymbol{\lambda} = \frac{\partial g}{\partial \boldsymbol{\Phi}}$

Gradient: $\frac{dg}{d\boldsymbol{p}} = \frac{\partial g}{\partial \boldsymbol{p}} + \boldsymbol{\lambda}^T \left(\frac{\partial \boldsymbol{b}}{\partial \boldsymbol{p}} - \frac{\partial \mathcal{M}}{\partial \boldsymbol{p}} \boldsymbol{\Phi} \right)$



- Exploit reciprocity: Only one additional “adjoint” measurement needed
- In-situ measurements self-calibrate against real-world losses/detuning
- Real-time, gradient-based optimization without big data sets or neural training neural networks

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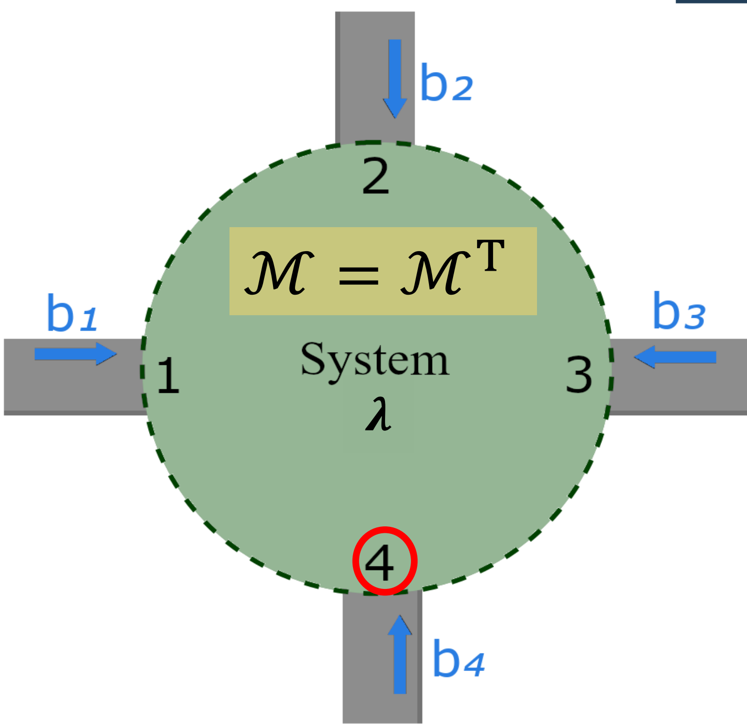

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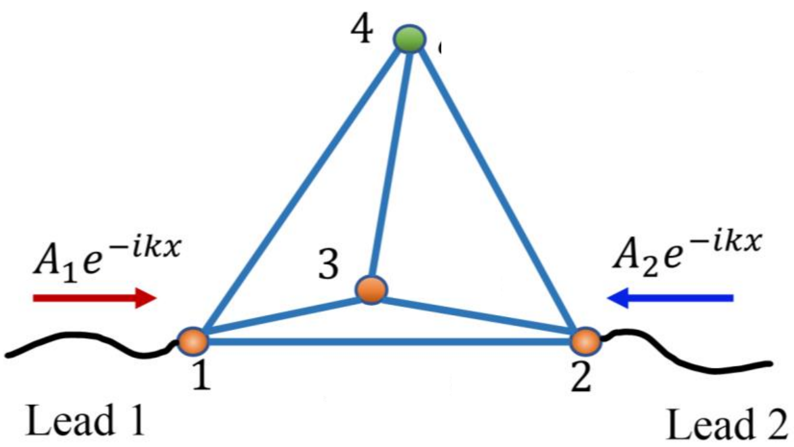
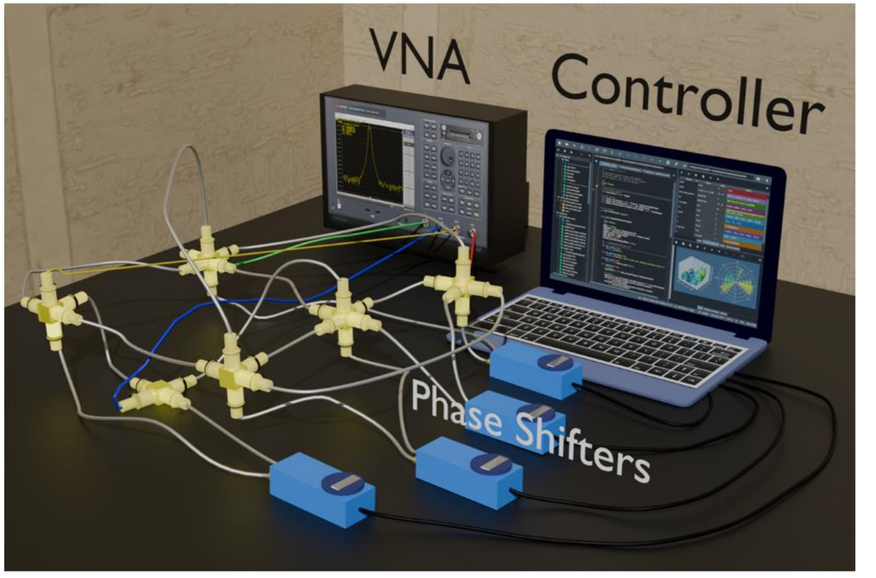
Microwave Graph Network





- Wave equation on each bond:

$$\frac{d^2}{dx_{mn}^2}\psi_{mn}^{(\alpha)} + k^2\psi_{mn}^{(\alpha)} = 0$$

- Wave continuity on each vertex:

$$\psi_{mn}^{(\alpha)}(x_{mn} = 0) = \phi_m^{(\alpha)}$$

$$\psi_{nm}^{(\alpha)}(x_{nm}) = \phi_n^{(\alpha)} \frac{\sin k(L_{nm} - x_{nm})}{\sin kL_{nm}} + \phi_m^{(\alpha)} \frac{\sin kx_{nm}}{\sin kL_{nm}}$$

- Current conservation:

$$\sum_n \frac{d\psi_{mn}^{(\alpha)}}{dx_{mn}}\Big|_{x_{mn}=0} + \sum_{\mu=1,2} \delta_{\mu,\alpha} \frac{d\psi_\mu^{(\alpha)}}{dx}\Big|_{x=0} = 0$$

- Matrix equation for the wave on the vertices

$$(M + iW^T W)\Phi^{(\alpha)} = 2\mathrm{i}\mathrm{W}^{\mathrm{T}}\mathrm{I}^{(\alpha)}$$

$$M_{mn} = \begin{cases} -\sum_{\gamma \neq m} \mathcal{A}_{m\gamma} \cot kL_{m\gamma}, m = n \\ \mathcal{A}_{mn} \csc kL_{mn}, m \neq n \end{cases}$$

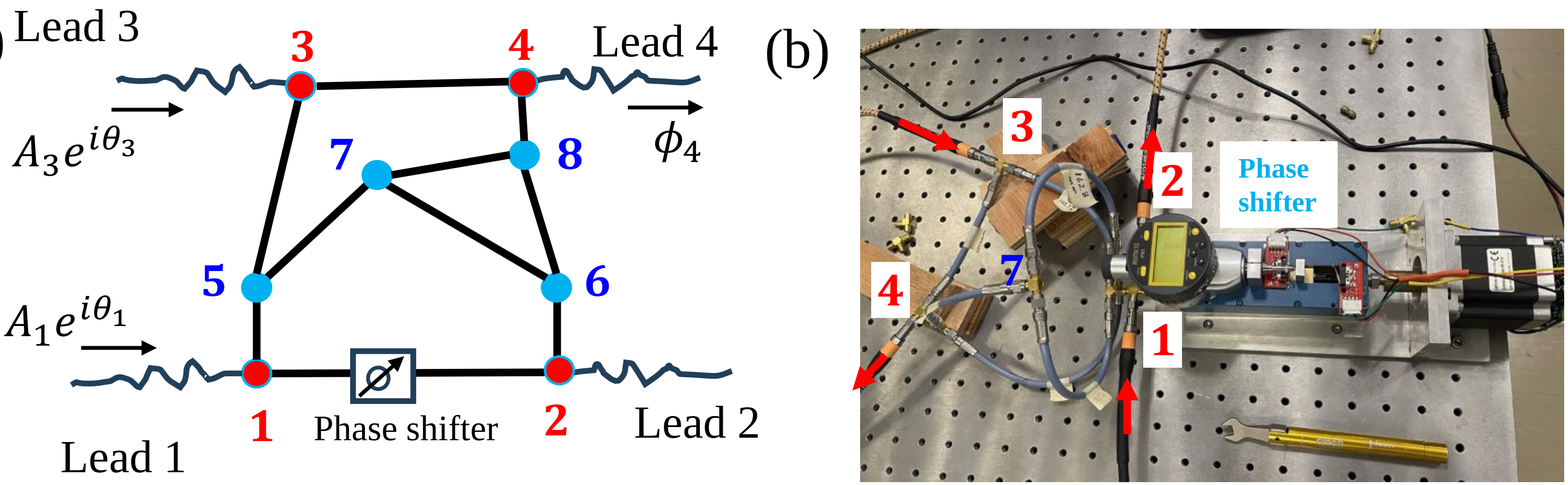
$$W = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \end{pmatrix}$$

$$I_n^{(\alpha)} = A_\alpha \cdot \delta_{\alpha,n}$$

T. Kottos and U. Smilansky, PRL, (1997)
O. Hul et al., Phys. Rev. E 69, 056205 (2004)
B. Dietz et al., Phys. Rev. E 95, 052202 (2017)

Example Modality 1- Targeted Mode Transport

(a) (b)

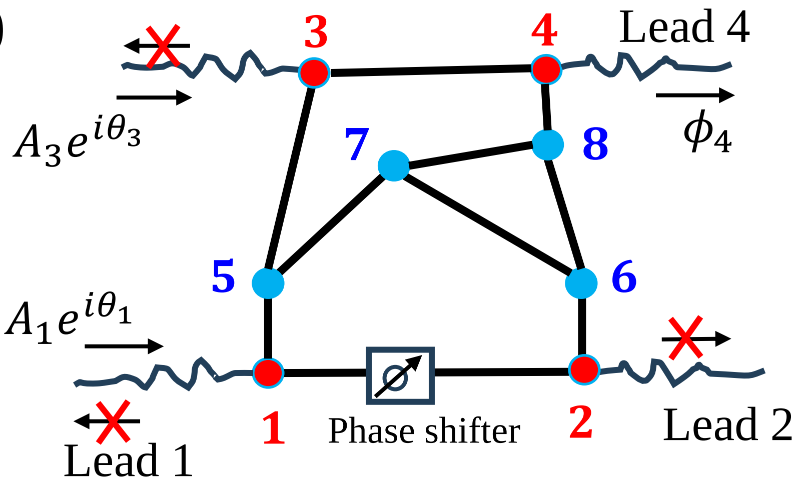


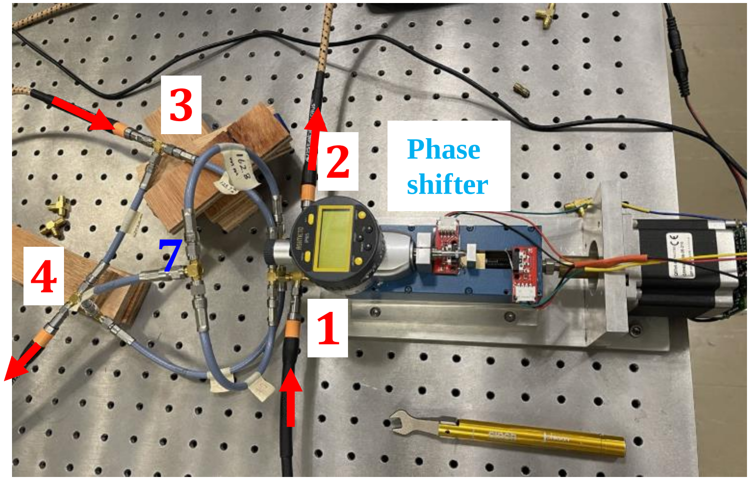
- Wave input from lead 1 and lead 3. Maximize transmittance at targeted port 4, i.e., maximize the objective function: $g = \frac{|\phi_4|^2}{A_1^2+A_3^2} = \frac{\phi_4\phi_4^*}{A_1^2+A_3^2}$, where ϕ_4 is the wave on vertex 4

- Three control parameters $\boldsymbol{p} = [\mathrm{L}_{12}, \mathrm{A}_3, \theta_3]$: length L_{12} tuned by phase shifter, input wave amplitude A_3 and phase θ_3.

- Gradient based optimization requires knowledge of $\frac{dg}{d\boldsymbol{p}} = [\frac{dg}{dL_{12}}, \frac{dg}{dA_3}, \frac{dg}{d\theta_3}]$.

Gradient Measurement: Adjoint Method and Finite Difference Method

(a)


(b)


- Digital Twin: calculate the gradient, based on modeling (CPU costly/model inaccurate).
- Finite difference method requires derivatives for each of the M parameters x at x_0: $\frac{\partial g}{\partial x}|_{x=x_0} = \frac{g(x_0+\delta)-g(x_0-\delta)}{2\delta}$, i.e., 2M measurements and 3M operations (time costly/noise sensitive)

Our goal is

(1) Get the gradient/derivative by *in-situ* measurement instead of solving the system equations;

(2) Minimize the number of measurements/operations (fast/noise resilient).

Implementation of Adjoint Method

Optimize the objective function: $g(\boldsymbol{p}, \Phi(\boldsymbol{p}), \Phi^*(\boldsymbol{p})) = \dfrac{|\phi_4|^2}{A_1^2 + A_3^2} = \dfrac{\phi_4 \phi_4^*}{A_1^2 + A_3^2}$

$$\frac{dg}{d\boldsymbol{p}} = \frac{\partial g}{\partial \boldsymbol{p}} + \frac{\partial g}{\partial \Phi}\frac{\partial \Phi}{\partial \boldsymbol{p}} + \frac{\partial g}{\partial \Phi^*}\frac{\partial \Phi^*}{\partial \boldsymbol{p}} = \frac{\partial g}{\partial \boldsymbol{p}} + \begin{bmatrix}\dfrac{\partial g}{\partial \Phi} & \dfrac{\partial g}{\partial \Phi^*}\end{bmatrix}\begin{bmatrix}\dfrac{\partial \Phi}{\partial \boldsymbol{p}} \\ \dfrac{\partial \Phi^*}{\partial \boldsymbol{p}}\end{bmatrix} \leftarrow ?$$

- The system wave equation:

$$(M + iW^T W)\Phi = 2\mathrm{iW^T I_{in}} \qquad M_{mn} = \begin{cases} -\sum_{\gamma \neq m} \mathcal{A}_{m\gamma} \cot kL_{m\gamma} + \lambda_m k, & m = n \\ \mathcal{A}_{mn} \csc kL_{mn}, & m \neq n \end{cases}$$

or written as: $f = (M + iW^T W)\Phi - 2\mathrm{iW^T I_{in}} = 0$

And the conjugate system equation: $f^* = (M^* - iW^T W)\Phi^* + 2\mathrm{iW^T I_{in}^*} = 0$

- Evaluation of $\dfrac{\partial \Phi}{\partial \boldsymbol{p}}$ and $\dfrac{\partial \Phi^*}{\partial \boldsymbol{p}}$ requires solution of two coupled algebraic equations

$$\frac{df}{d\boldsymbol{p}} = \frac{\partial f}{\partial \boldsymbol{p}} + \frac{\partial f}{\partial \Phi}\frac{\partial \Phi}{\partial \boldsymbol{p}} + \frac{\partial f}{\partial \Phi^*}\frac{\partial \Phi^*}{\partial \boldsymbol{p}} = 0$$

$$\frac{df^*}{d\boldsymbol{p}} = \frac{\partial f^*}{\partial \boldsymbol{p}} + \frac{\partial f^*}{\partial \Phi}\frac{\partial \Phi}{\partial \boldsymbol{p}} + \frac{\partial f^*}{\partial \Phi^*}\frac{\partial \Phi^*}{\partial \boldsymbol{p}} = 0$$

Known →

$$\begin{pmatrix}\dfrac{\partial \Phi}{\partial \boldsymbol{p}} \\ \dfrac{\partial \Phi^*}{\partial \boldsymbol{p}}\end{pmatrix} = -\begin{pmatrix}\dfrac{\partial f}{\partial \Phi} & \dfrac{\partial f}{\partial \Phi^*} \\ \dfrac{\partial f^*}{\partial \Phi} & \dfrac{\partial f^*}{\partial \Phi^*}\end{pmatrix}^{-1}\begin{pmatrix}\dfrac{\partial f}{\partial \boldsymbol{p}} \\ \dfrac{\partial f^*}{\partial \boldsymbol{p}}\end{pmatrix}$$

Implementation of Adjoint Method

$(1 \times 2N)$ $(2N \times 2N)$ $(2N \times M)$

$$\frac{dg}{d\boldsymbol{p}} = \frac{\partial g}{\partial \boldsymbol{p}} + \begin{bmatrix} \frac{\partial g}{\partial \Phi} & \frac{\partial g}{\partial \Phi^*} \end{bmatrix} \begin{bmatrix} \frac{\partial \Phi}{\partial \boldsymbol{p}} \\ \frac{\partial \Phi^*}{\partial \boldsymbol{p}} \end{bmatrix} = \frac{\partial g}{\partial \boldsymbol{p}} - \begin{bmatrix} \frac{\partial g}{\partial \Phi} & \frac{\partial g}{\partial \Phi^*} \end{bmatrix} \begin{pmatrix} \frac{\partial f}{\partial \Phi} & \frac{\partial f}{\partial \Phi^*} \\ \frac{\partial f^*}{\partial \Phi} & \frac{\partial f^*}{\partial \Phi^*} \end{pmatrix}^{-1} \begin{pmatrix} \frac{\partial f}{\partial \boldsymbol{p}} \\ \frac{\partial f^*}{\partial \boldsymbol{p}} \end{pmatrix}$$

$$= \lambda^T = (\lambda_1, \lambda_1^*)^T$$

What is the physical meaning of λ?

$$\begin{pmatrix} \left(\frac{\partial f}{\partial \Phi}\right)^T & \left(\frac{\partial f}{\partial \Phi^*}\right)^T \\ \left(\frac{\partial f^*}{\partial \Phi}\right)^T & \left(\frac{\partial f^*}{\partial \Phi^*}\right)^T \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_1^* \end{pmatrix} = \begin{pmatrix} -\left(\frac{\partial g}{\partial \Phi}\right)^T \\ -\left(\frac{\partial g}{\partial \Phi^*}\right)^T \end{pmatrix}$$

$M + iW^TW$ 0 0 $M^* - iW^TW$ $2\mathrm{iW^TI_{adj}}$ $-2\mathrm{iW^TI_{adj}^*}$

λ_1 is the solution of the Maxwell's Equation for a certain excitation I_{adj}:

$$\mathrm{I_{adj}} = \left[0, \quad 0, \quad 0, \quad \frac{i}{2}\frac{\phi_4^*}{A_1^2 + A_3^2}\right]$$

Reminder: $f = (M + iW^TW)\Phi - 2\mathrm{iW^TI_{in}}$

Implementation of *in situ* Adjoint Method

- Forward Equation:

The same for time-reversal invariance systems

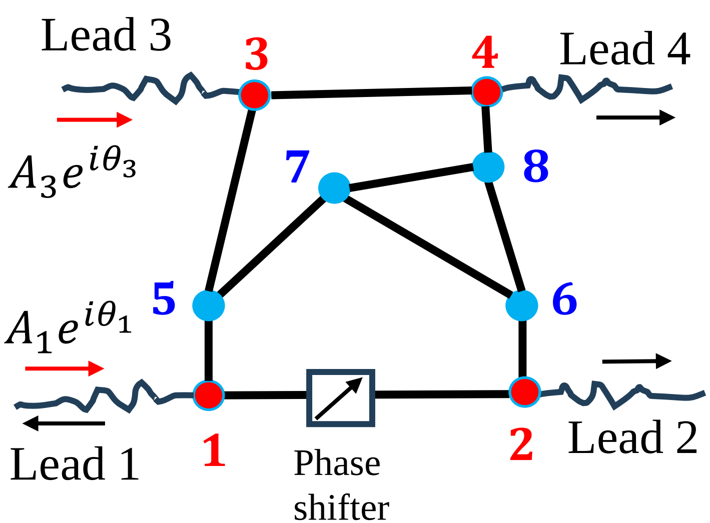
$$(M + iW^T W)\Phi = 2\mathrm{i}\mathrm{W}^{\mathrm{T}}\mathrm{I}_{\mathrm{in}}$$

$$\mathrm{I}_{\mathrm{in}} = [A_1 e^{i\theta_1}, \quad 0, \quad A_3 e^{i\theta_3}, \quad 0]$$

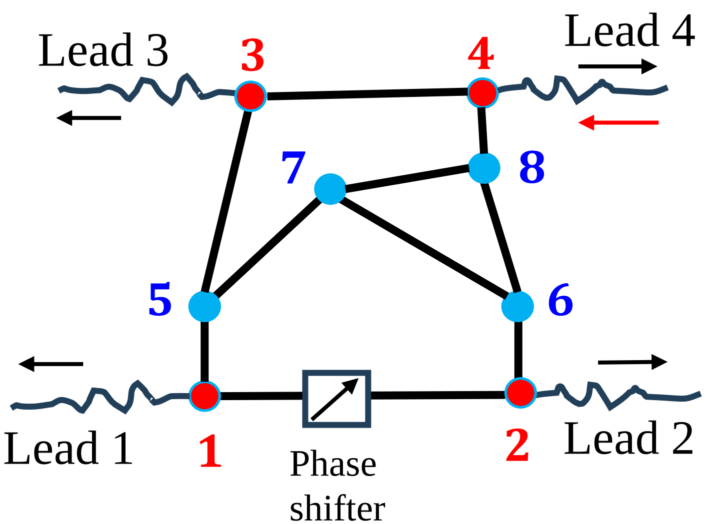
- Adjoint (backward) Equation:

$$(M + iW^T W)\lambda_1 = 2\mathrm{i}\mathrm{W}^{\mathrm{T}}\mathrm{I}_{\mathrm{adj}}$$

$$\mathrm{I}_{\mathrm{adj}} = \left[0, \quad 0, \quad 0, \quad \frac{i}{2}\frac{\phi_4^*}{A_1^2 + A_3^2}\right]$$



Forward



Backward

Instead of doing matrix inversion via computation, we could directly measure the forward and adjoint field, this is *in situ* adjoint method.

Implementation of *in situ* Adjoint Method

$$\frac{dg}{d\boldsymbol{p}} = \frac{\partial g}{\partial \boldsymbol{p}} + \left(\lambda^T, \lambda^{*T}\right)\begin{pmatrix} \frac{\partial f}{\partial \boldsymbol{p}} \\ \frac{\partial f^*}{\partial \boldsymbol{p}} \end{pmatrix}$$

Backward measurement (λ^T, λ^{*T})

Forward measurement ($\frac{\partial f}{\partial \boldsymbol{p}}, \frac{\partial f^*}{\partial \boldsymbol{p}}$)

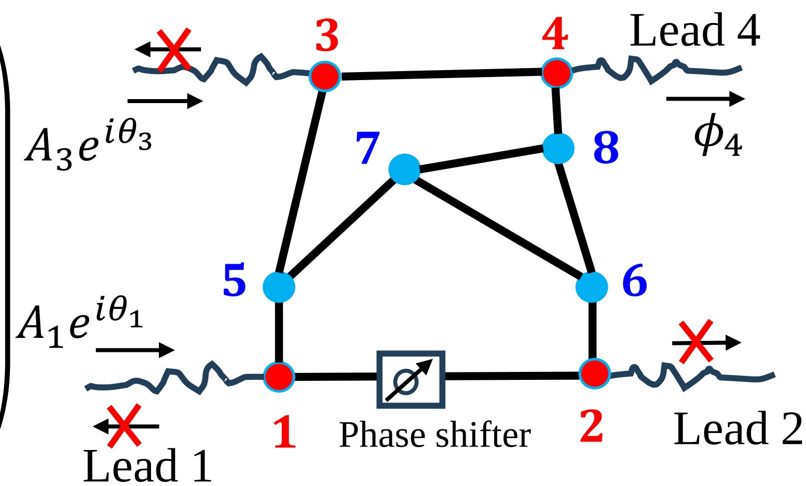
Reminder:

$$f = (M + iW^TW)\Phi - 2\mathrm{i}\mathrm{W}^\mathrm{T}\mathrm{I}_{\mathrm{in}}$$

$$\boldsymbol{p} = [\mathrm{L}_{12}, \mathrm{A}_3, \theta_3]$$

Measure twice: forward and backward (adjoint) waves, avoid the matrix inversion!

$$\frac{\partial f}{\partial \boldsymbol{p}} = \begin{pmatrix} \frac{k\phi_1}{\sin^2 kL_{12}} - \frac{k\cos kL_{12}\,\phi_2}{\sin^2 kL_{12}} & 0 & 0 \\ \frac{k\phi_2}{\sin^2 kL_{12}} - \frac{k\cos kL_{12}\,\phi_1}{\sin^2 kL_{12}} & 0 & 0 \\ 0 & -2ie^{i\theta_3} & iA_3e^{i\theta_3} \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{pmatrix}$$



- We only need to measure the vertices local to our parameters!
- Tuning the system locally can change the system dramatically due to wave chaos!

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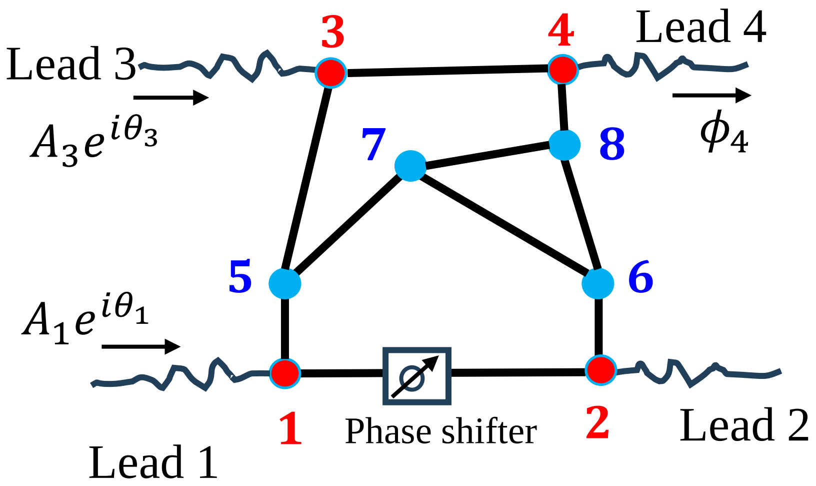
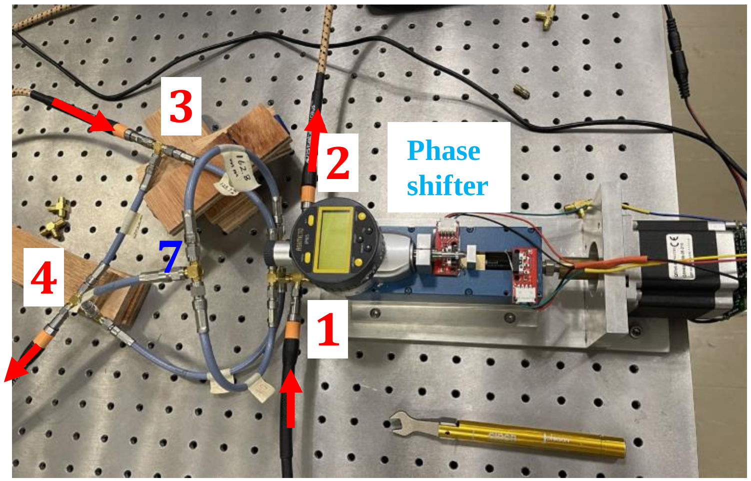
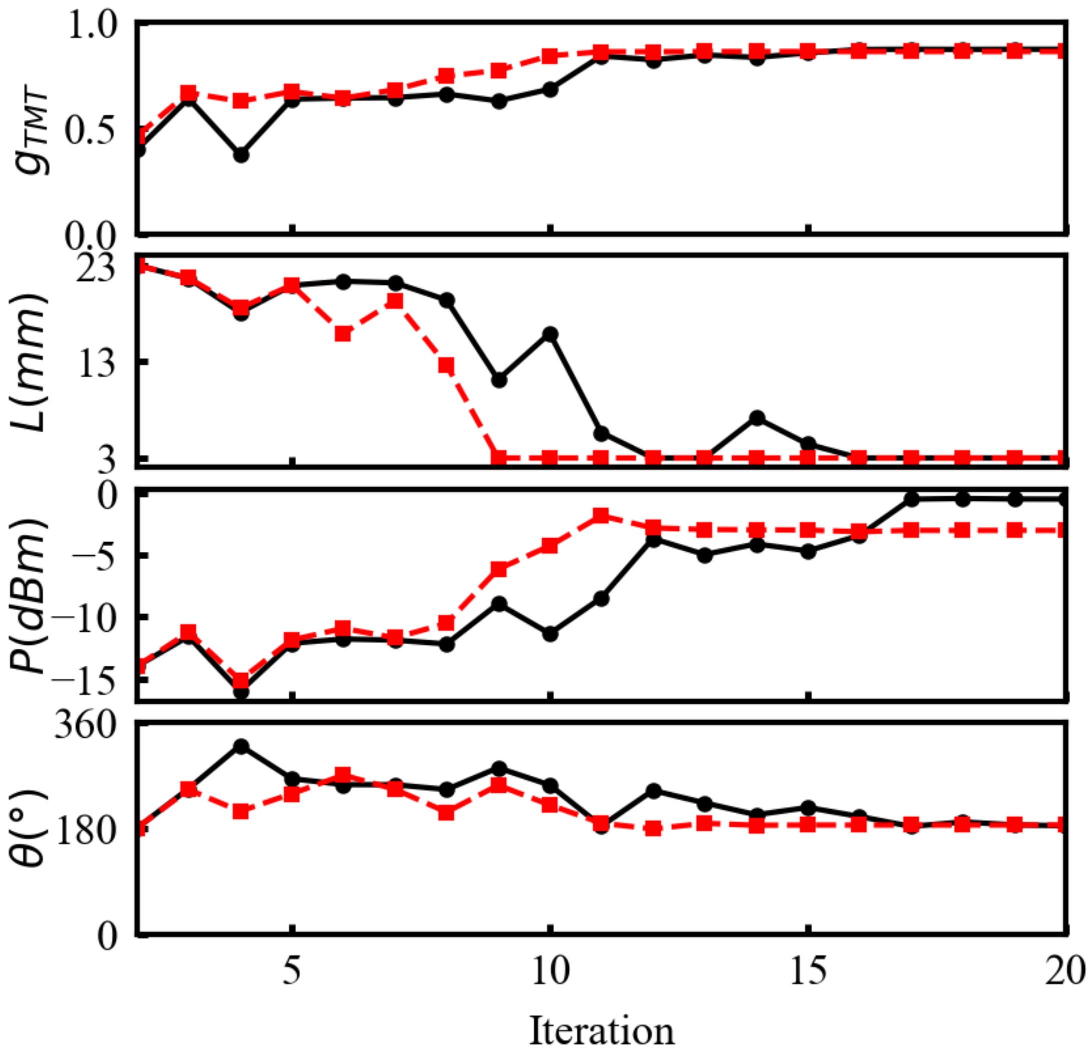
Example Modality 1: Targeted Mode Transmission





1. Forward Measurement
   - Inject signals into the system and measure the forward response.
2. Adjoint Measurement
   - Construct input based on the forward step and measure the adjoint response.
3. Gradient Calculation
   - Use measured forward and adjoint fields to calculate $\frac{dg}{dp}$
4. Parameter Update
   - Apply gradient-based optimization (e.g. MMA) and adjust phase shifter length and relative phase/amplitude
5. Iteration & Convergence
   - Repeat steps 1-4 until the objective function reaches a local min/max.

Example Modality 1: Targeted Mode Transmission

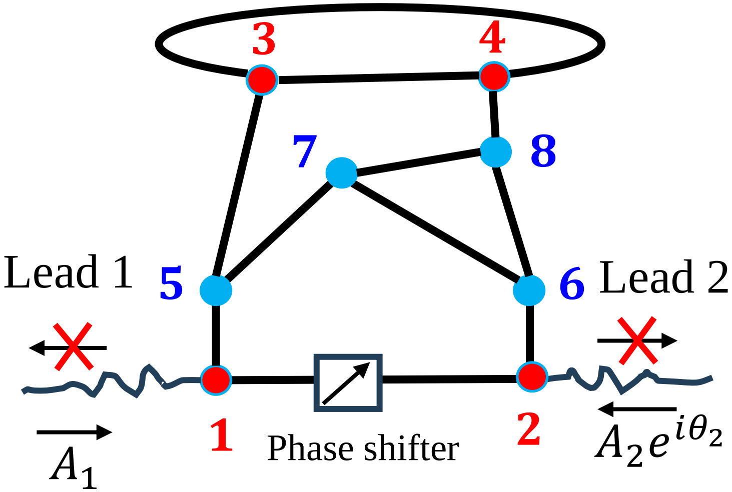


Maximize the objective function:

$$g = \frac{|\phi_4|^2}{A_1^2 + A_3^2} = \frac{\phi_4 \phi_4^*}{A_1^2 + A_3^2}$$

	In situ adjoint	Digital twin
Objective: g	0.873	0.862
L(mm)	3.00	3.00
Power(dBm)	-0.47	-2.99
Phase(°)	184.6	185.1

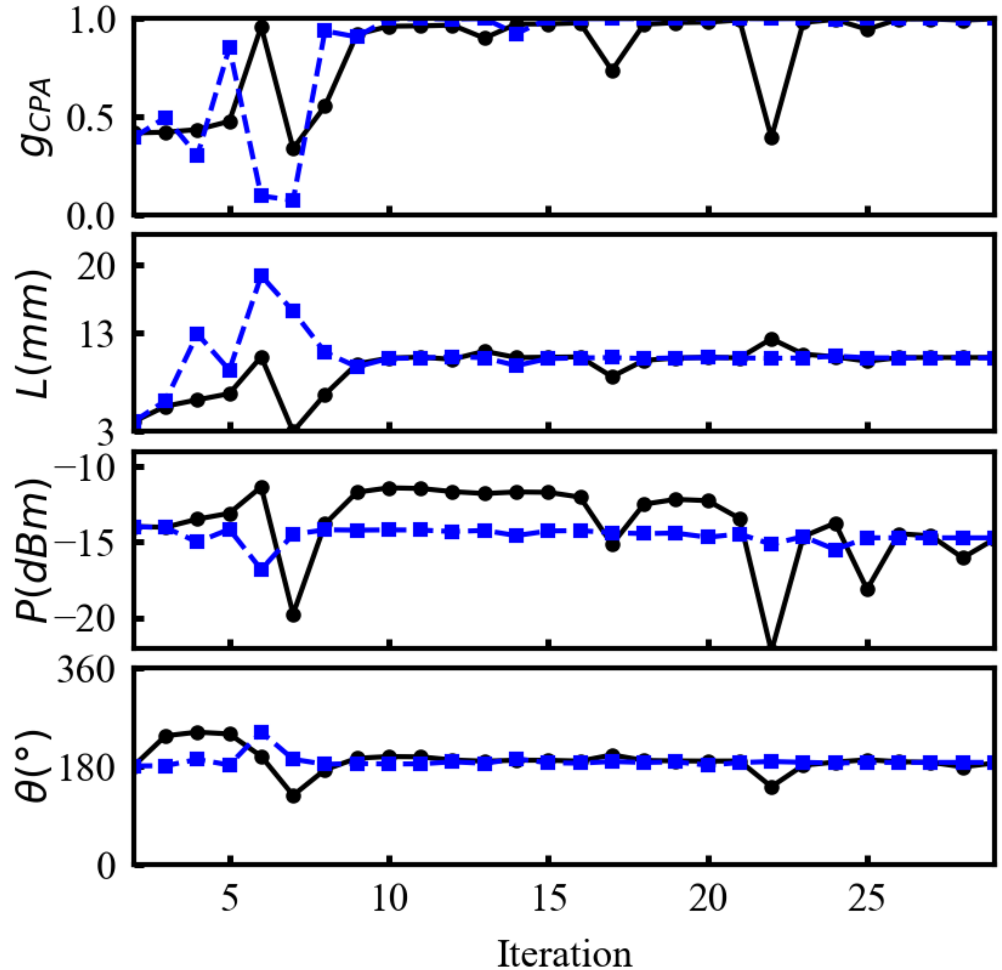
Example Modality 2: Coherent Perfect Absorption



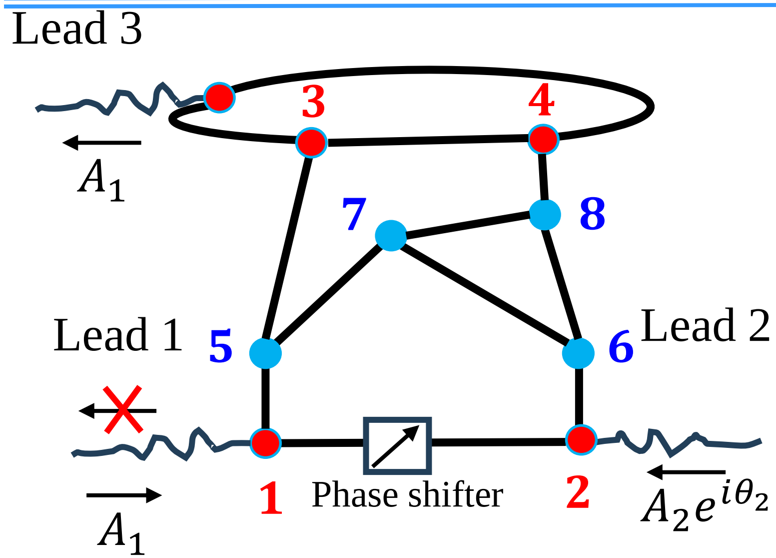
Maximize the objective function (absorption):

$$g = 1 - \frac{|\phi_1 - A_1|^2 + |\phi_2 - A_2 e^{i\theta_2}|^2}{A_1^2 + A_2^2}$$

	In situ adjoint	Digital twin
Objective: g	0.999	0.999
L(mm)	10.52	10.48
Power(dBm)	-14.82	-14.72
Phase(°)	185.2	186.4



Example Modality 3: Invisibility



a1(dBm)	a1(DEG)
-15.0079	-90.5411

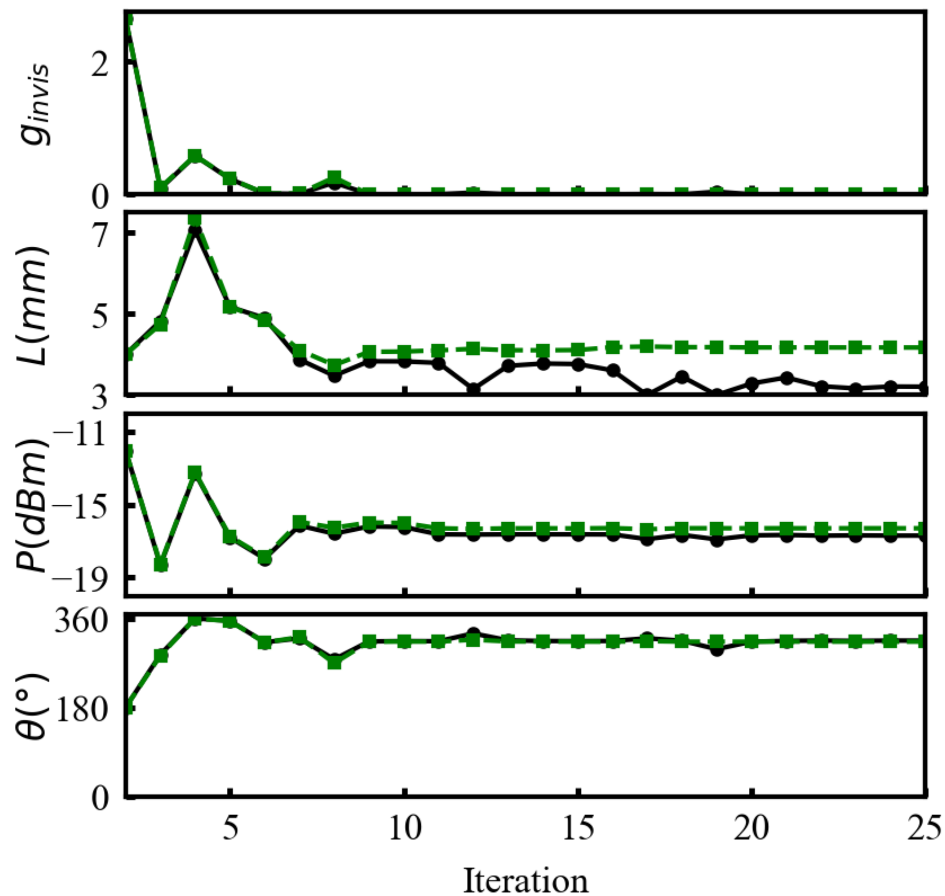
b3(dBm)	b3(DEG)
-14.9878	-90.6914

b1(dBm)
-66.7887

Minimize the objective function (difference):

$$g_{invis} = \frac{|\phi_3 - A_1|^2}{A_1^2} + \frac{|\phi_1 - A_1|^2}{A_1^2 + A_2^2}$$

	In situ adjoint	Digital twin
Objective: g	1.65e-5	4.73e-7
L(mm)	3.19	4.17
Power(dBm)	-16.71	-16.30
Phase(°)	315.1	313.0



Outline

- General Problem: Challenges in Wireless Communications
- Methodologies

1. Time-Reversal Mirror Protocol
2. Linear and Nonlinear Wavefront Shaping Protocols
3. Cavity Shaping Protocols

- In-Situ Adjoint Method

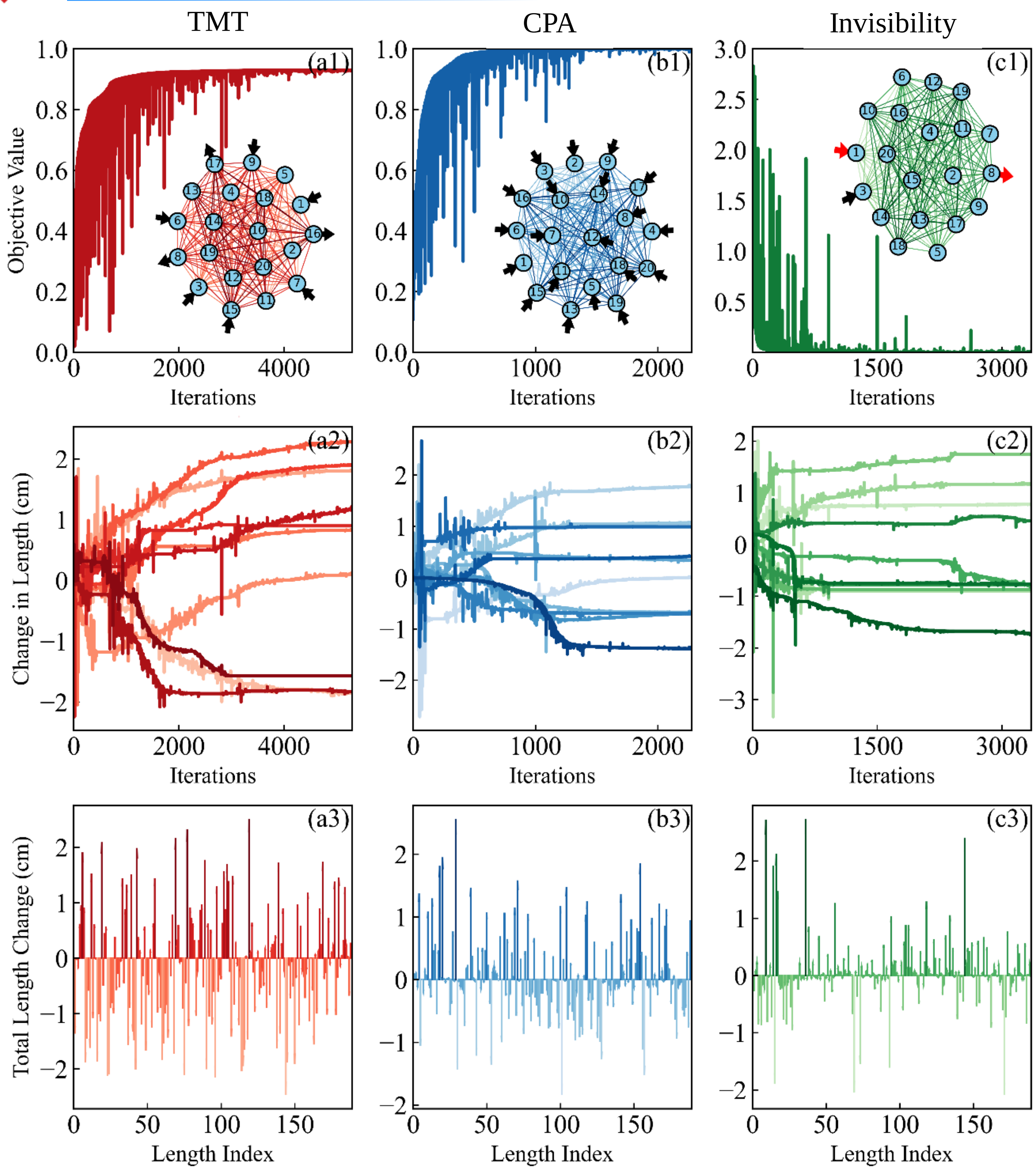
1. General Principles of Adjoint Method
2. Physical Implementation in Complex Networks

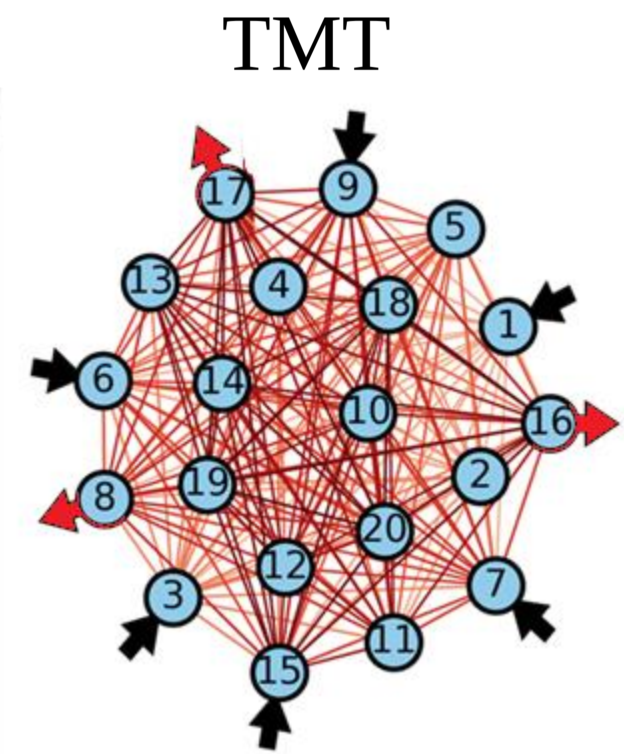
- In-Situ Modality Examples

1. Targeted Mode Transmission
2. Coherent Perfect Absorption
3. Invisibility

- In-Silico Generalization
- Outlook

In-Silico (simulations) Using Large Complex Networks





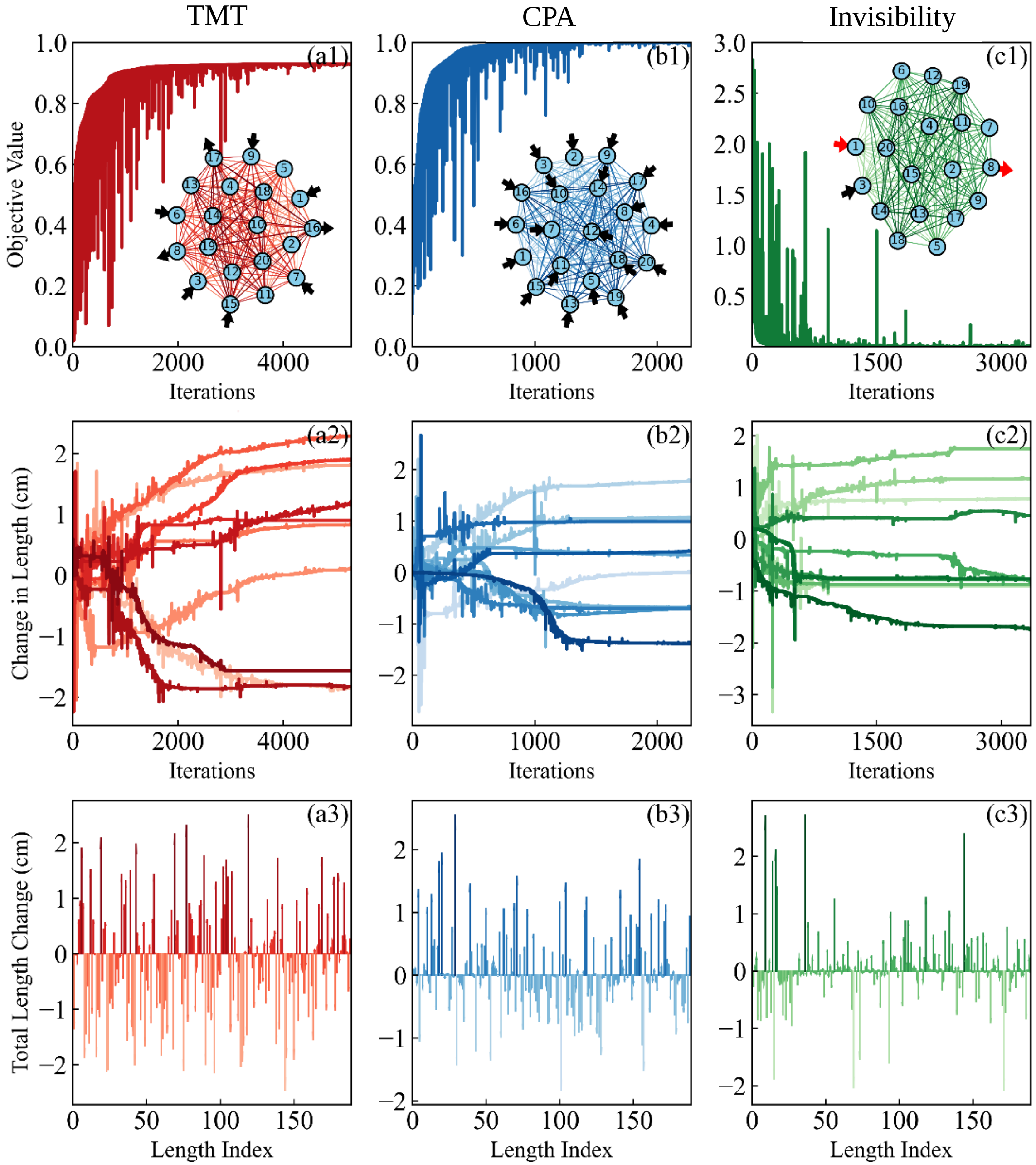
For a fully connected graph:

# of Bonds = $\frac{n(n-1)}{2} \approx n^2$,

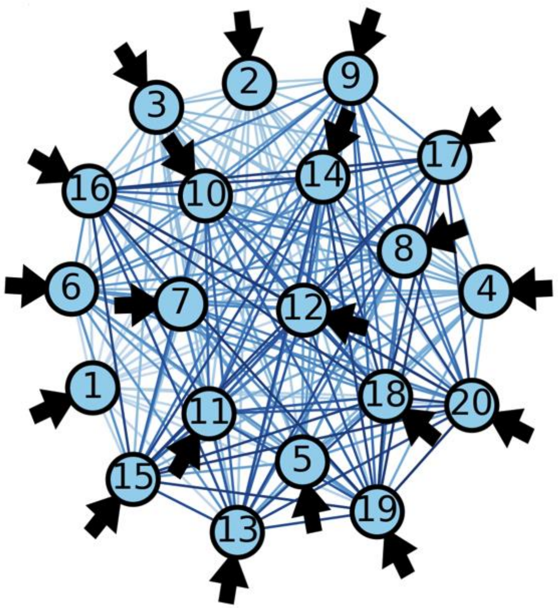
In-Situ Adjoint only requires n local measurements!

190 bonds → 20 measurements!

In-Silico (simulations) Using Large Complex Networks



CPA



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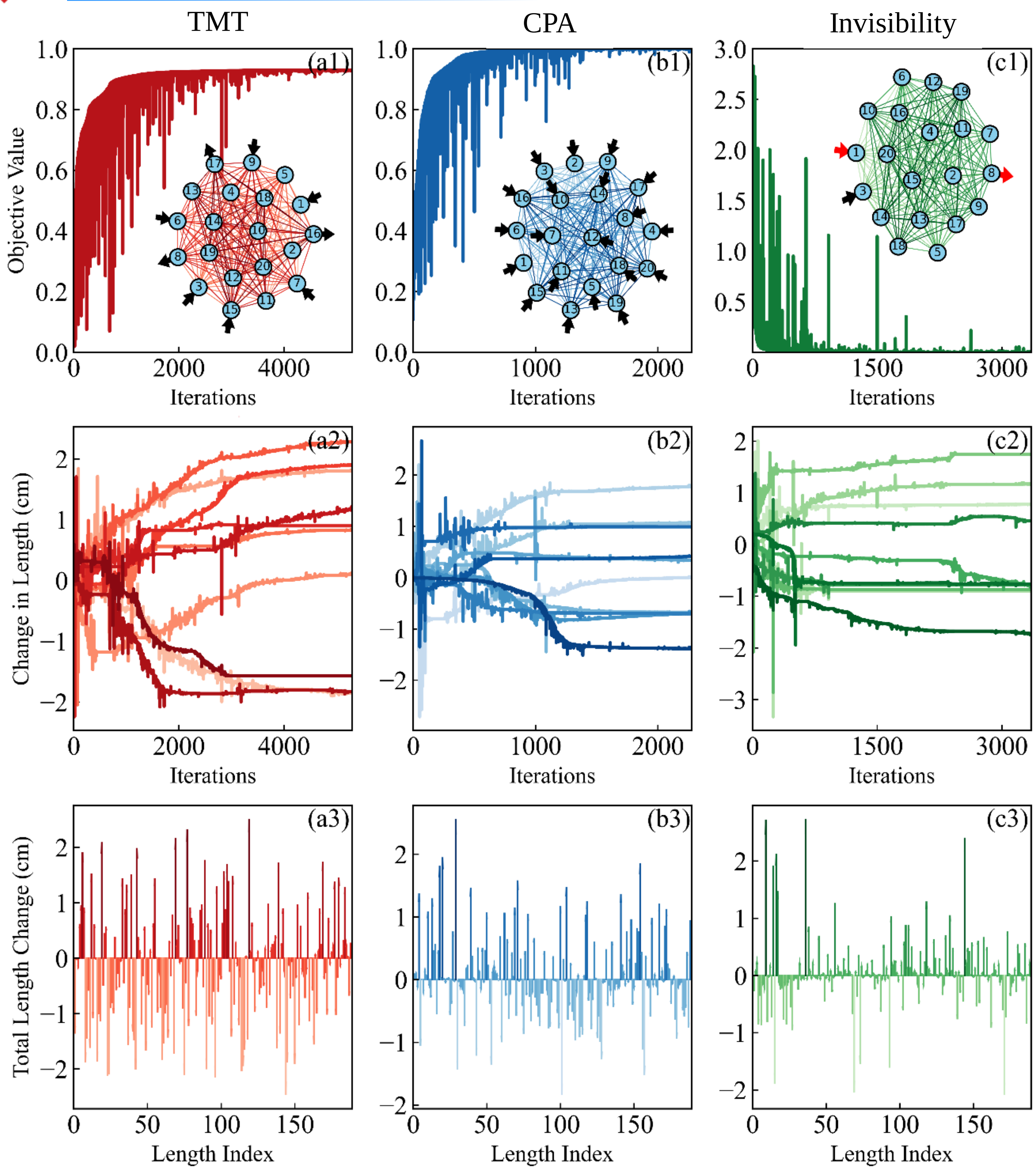
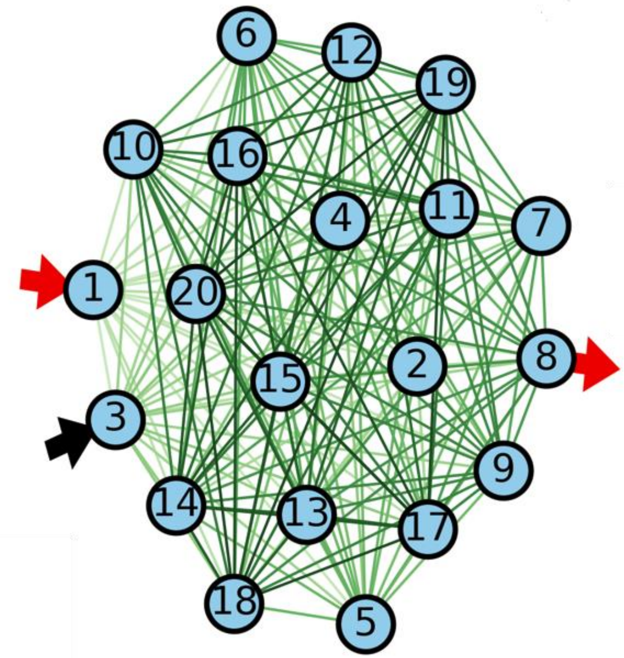


Invisibility



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