Wave Transport in Complex Systems
Department of Physics

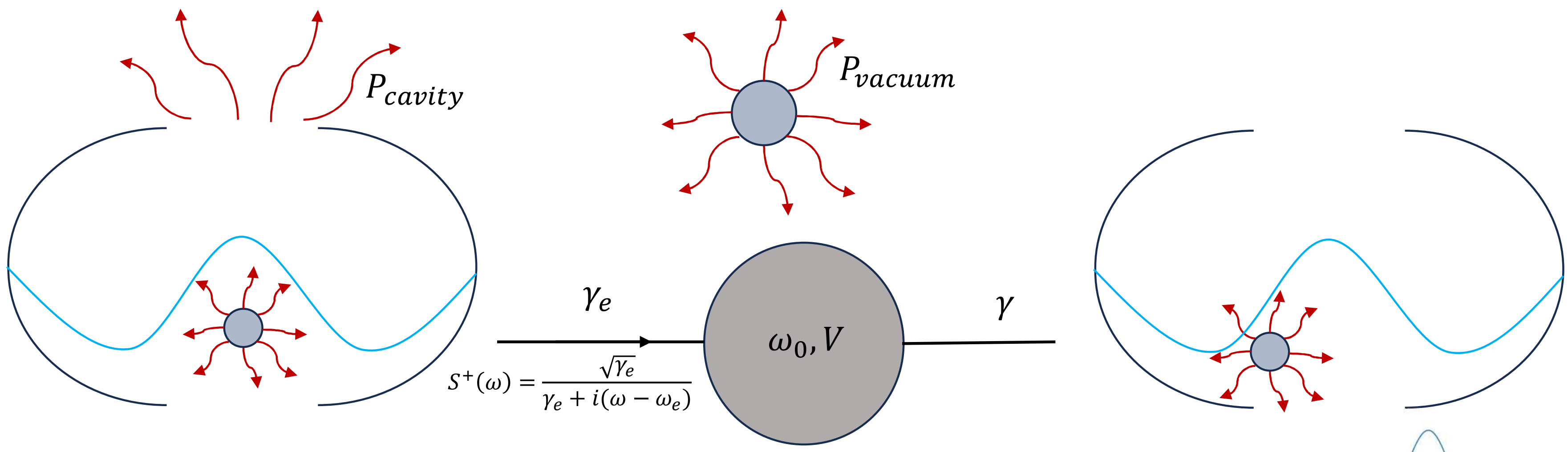
Enhanced Emission Near Exceptional Points Shaped by Riemann Surface Topology

Nicolas Wyszkowski

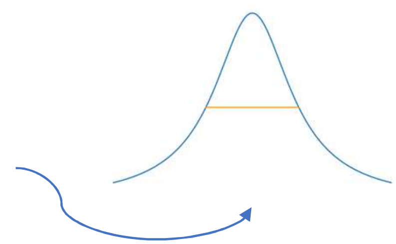
Arunn Suntharalingam
Lucas Fernández-Alcázar
Max Vitek

Tsampikos Kottos

Using Resonant Cavities to Modify Emission



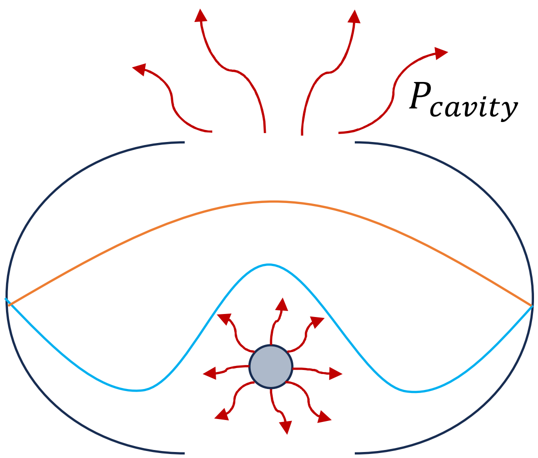
Purcell Enhancement Factor: $F_p = \frac{P_{cavity}}{P_{vacuum}} = \frac{3\pi c^3}{\omega_e^2 V(\gamma+\gamma_e)} \rightarrow F_p \propto Q$



Resonant cavities modify the spontaneous emission rate of sources inside them

Pick, A., Lin, Z., Jin, W., & Rodriguez, A. W. (2017). Enhanced nonlinear frequency conversion and Purcell enhancement at exceptional points. *Physical Review. B./Physical Review. B*, *96*(22)

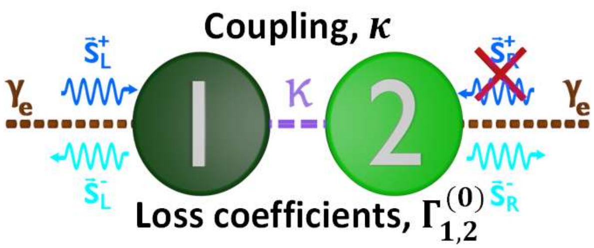
Using Resonant Cavities to Modify Emission



What about multiple modes?

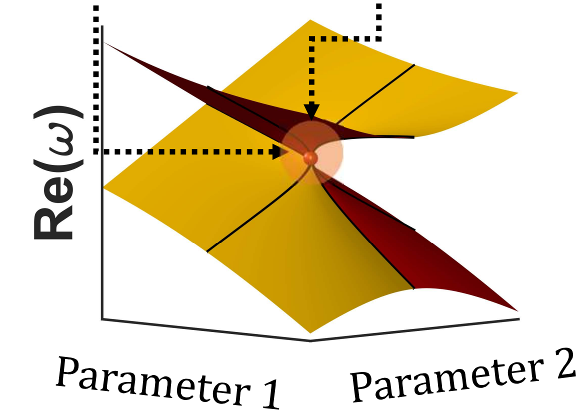
Exceptional Point Degeneracies (EPDs)

Non-Hermitian Degeneracies



Eigenvalues: DEGENERATE
Eigenvectors: DEGENERATE

EPD-2 Condition:
$\boldsymbol{\kappa_{EP} = |\Gamma_1 - \Gamma_2|/2}$



Expansion around degeneracy:

Newton-Puiseux Series:

$$\delta\omega = \sum_{n=1} c_n \varepsilon^{n/M}$$

$M \leq$ **order of degeneracy**

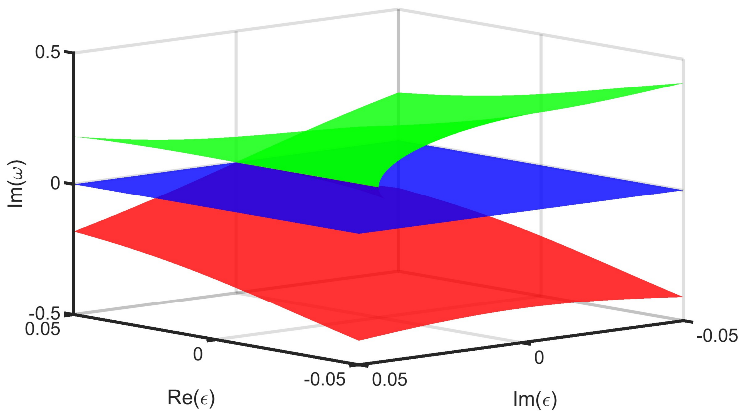
Typical expansion at an EPD-N:

$$\boldsymbol{\delta\omega = c_1 \varepsilon^{1/N} + \mathcal{O}\left(\varepsilon^{2/N}\right)}$$

Riemann Surface of Eigenfrequencies: EPD-3

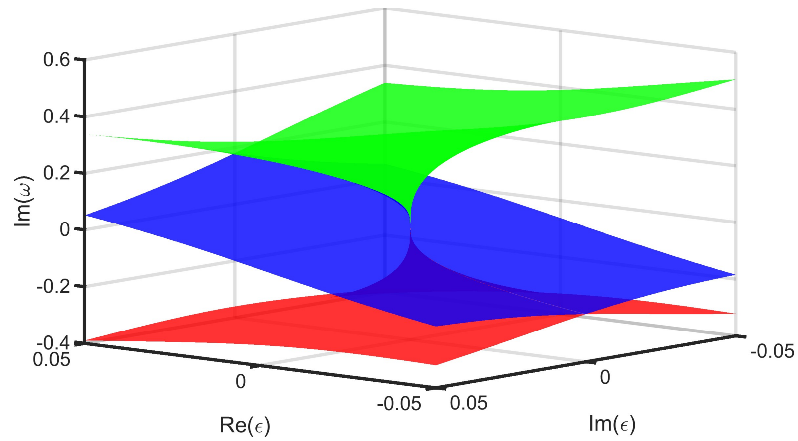
Square Root Expansion (SRE)

$$\delta\omega = a_1\varepsilon^{1/2} + \mathcal{O}(\varepsilon)$$



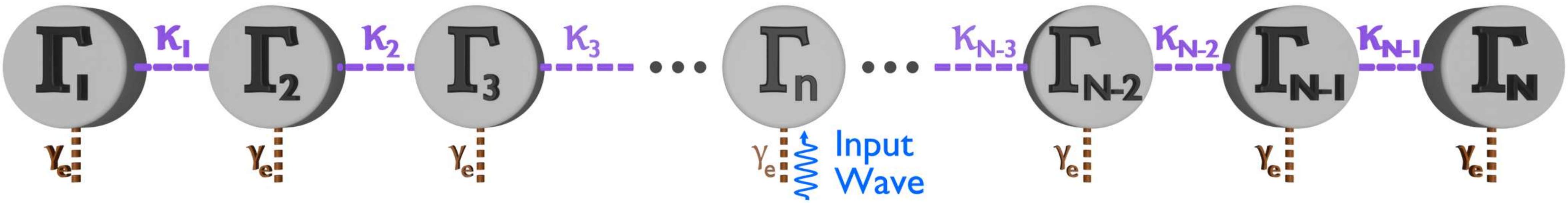
Cubic Root Expansion (CRE)

$$\delta\omega = b_1\varepsilon^{1/3} + \mathcal{O}(\varepsilon^{2/3})$$



Demange, G., & Graefe, E.-M. (2011). Signatures of three coalescing eigenfunctions. *Journal of Physics. A, Mathematical and Theoretical*, *45*(2), 025303

Relation Between Total Power and LDOS



LDOS: $\xi_n(\omega) = -\frac{1}{\pi}\Im(G_{nn}) = \frac{1}{\pi}((\Gamma_n+\gamma_e)|G_{nn}|^2 + \sum_{j\neq n}\Gamma_j\left|G_{jn}\right|^2)$

We connect with a scattering framework:

Transmittance: $T_{jn} = 4\gamma_e\Gamma_j\left|G_{jn}\right|^2 = P_{jn}/P_{input}$

→ $T_{total} = 4\pi\gamma_e(\xi_n(\omega) - \frac{1}{\pi}(\Gamma_n+\gamma_e)|G_{nn}|^2)$

$\Gamma_n + \gamma_e \ll \Gamma_j, \mathrm{j} \neq n$ → $T_{total} \approx 4\pi\gamma_e\xi_n(\omega)$

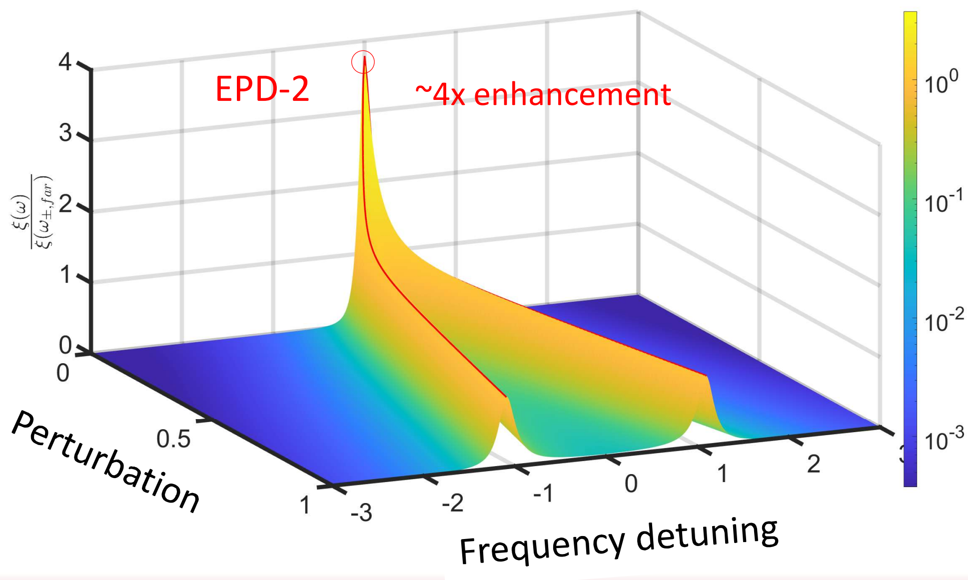
LDOS at an EPD-2

$$T_{total} \approx 4\pi\gamma_e\xi_1(\omega)$$

$$\pi\xi_{1,EP}(\omega) \equiv -\Im(G_{11}(\omega;\kappa_{EP})) = \underbrace{\frac{\bar{\Gamma}^2(\Gamma_2-\Gamma_1)}{[(\omega-\omega_0)^2+\bar{\Gamma}^2]^2}}_{\text{Lorentzian-squared}} + \underbrace{\frac{\Gamma_1}{(\omega-\omega_0)^2+\bar{\Gamma}^2}}_{\text{Lorentzian}}$$

Normalized LDOS in Passive Dimer with EPD-2

Theoretical bound: 4x enhancement



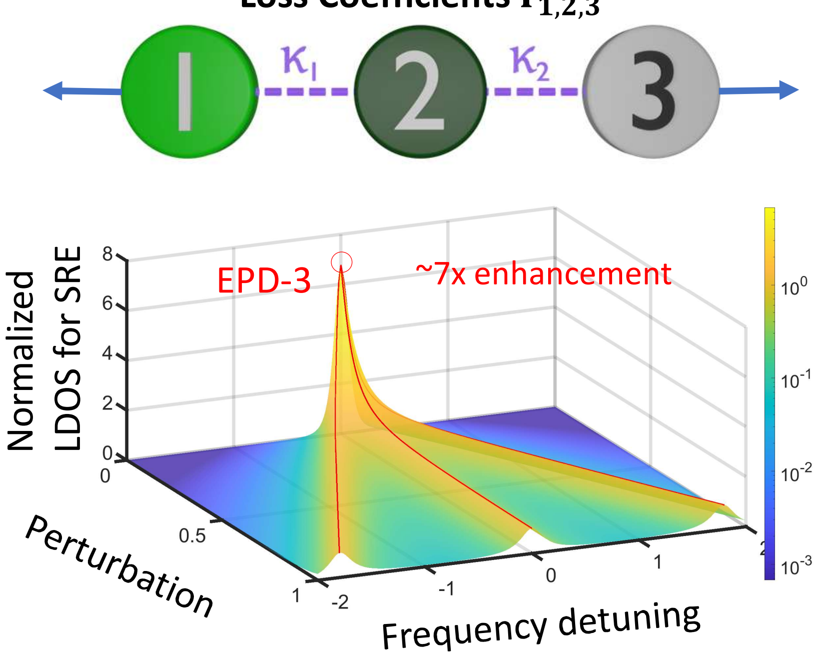
LDOS at an EPD-3

$$T_{total} \approx 4\pi\gamma_e \xi_3(\omega)$$

$$\pi\xi_{3,EP}(\omega) \equiv \Im\big(G_{33}(\omega;\kappa_{1,EP};\kappa_{2,EP})\big) = \underbrace{\frac{f(\Gamma_1,\Gamma_2)}{[(\omega-\omega_0)^2+\bar{\Gamma}^2]^3}}_{\text{Lorentzian-cubed}} + \underbrace{\frac{g(\Gamma_1,\Gamma_2)}{[(\omega-\omega_0)^2+\bar{\Gamma}^2]^2}}_{\text{Lorentzian-squared}} + \underbrace{\frac{\Gamma_3}{(\omega-\omega_0)^2+\bar{\Gamma}^2}}_{\text{Lorentzian}}$$

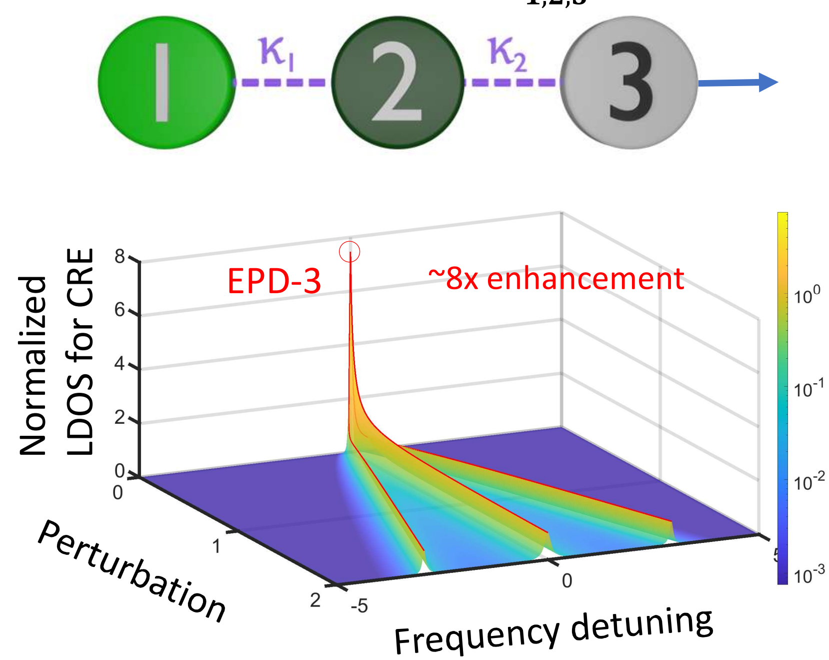
Square Root Expansion (SRE)

Loss Coefficients $\Gamma_{1,2,3}$



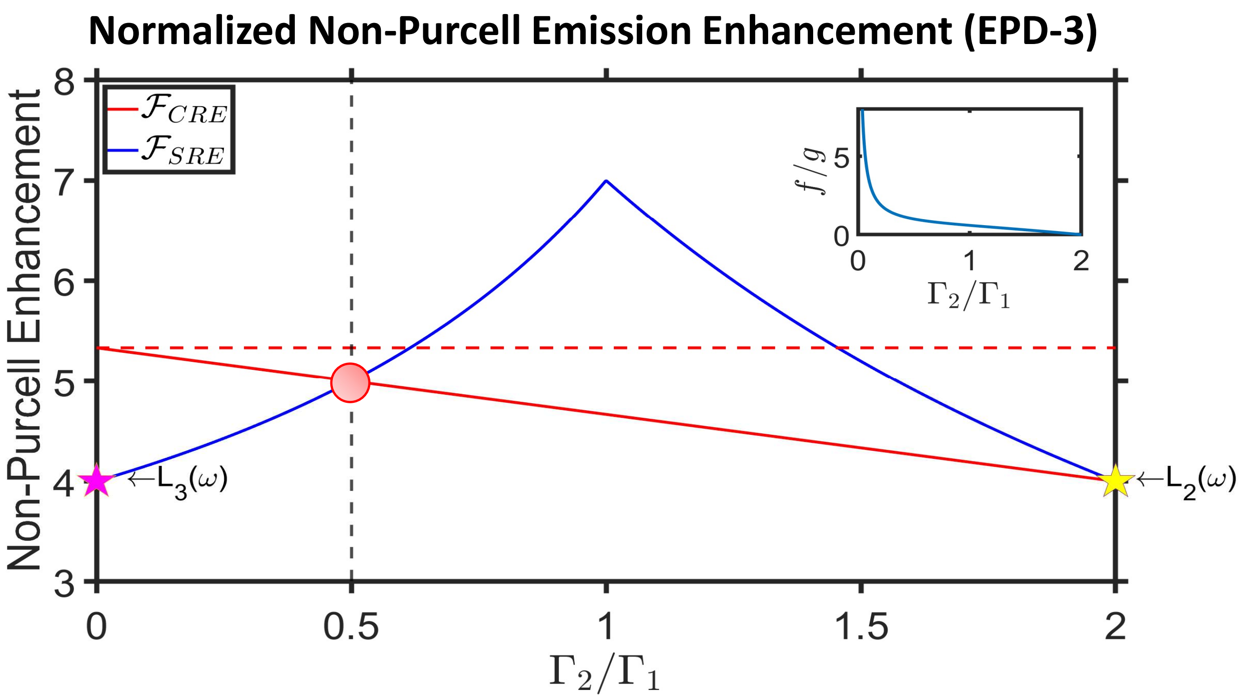
Cubic Root Expansion (CRE)

Loss Coefficients $\Gamma_{1,2,3}$

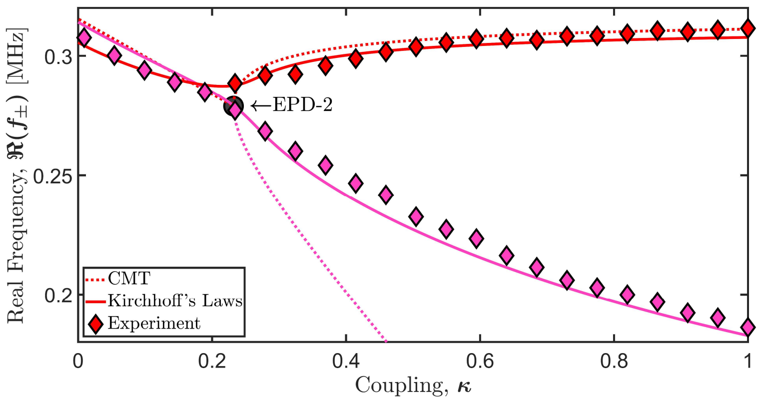


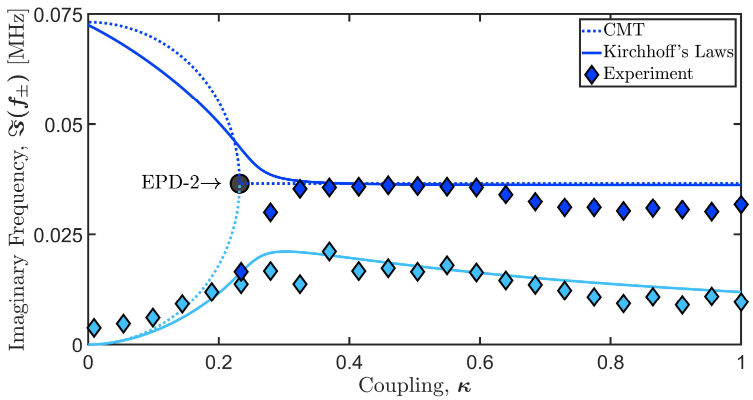
Non-Purcell Emission Enhancement (EPD-3)

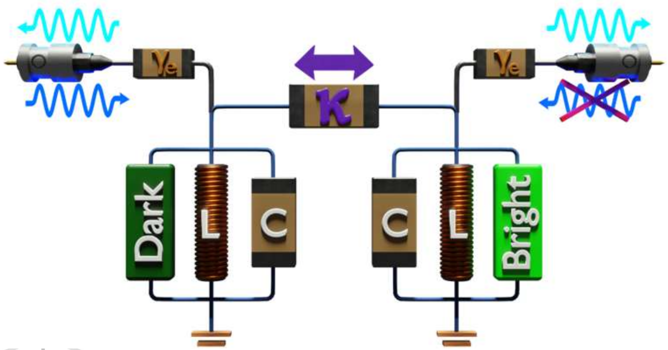
Purcell Enhancement Factor: $F_p \propto Q$ → **Non-Purcell Enhancement Factor**: $\boldsymbol{\mathcal{F}} \equiv \frac{P_{total}(\omega_0;\kappa_{EP})/Q_{EP}}{P_{total}(\omega_{max};\kappa_\infty)/Q_\infty} = \frac{\xi(\omega_0;\kappa_{EP})}{\xi(\omega_{max};\kappa_\infty)} \cdot \frac{Q_\infty}{Q_{EP}}$

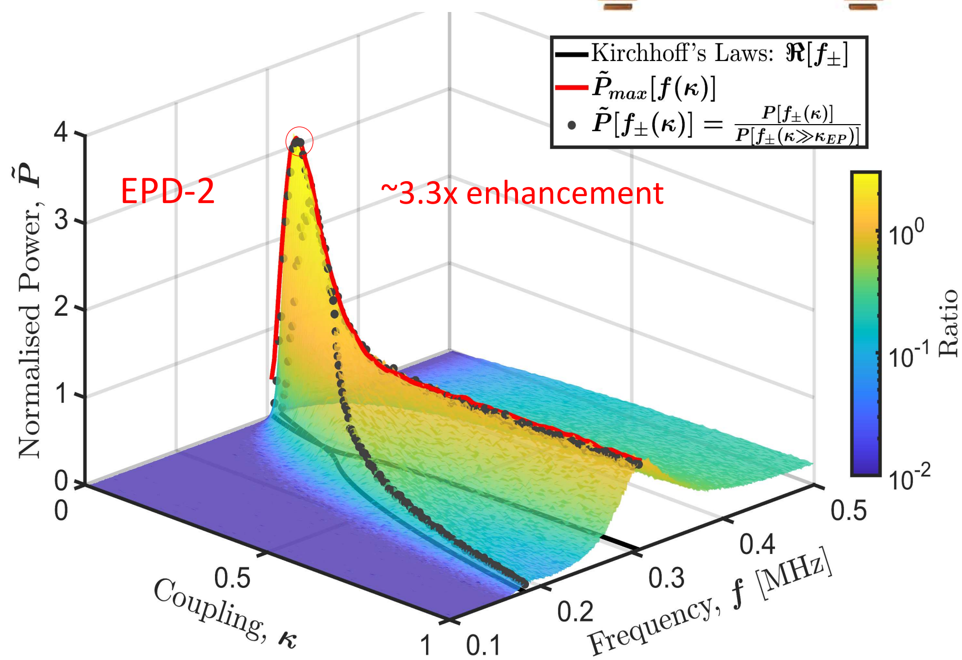


Dimer Experimental Results

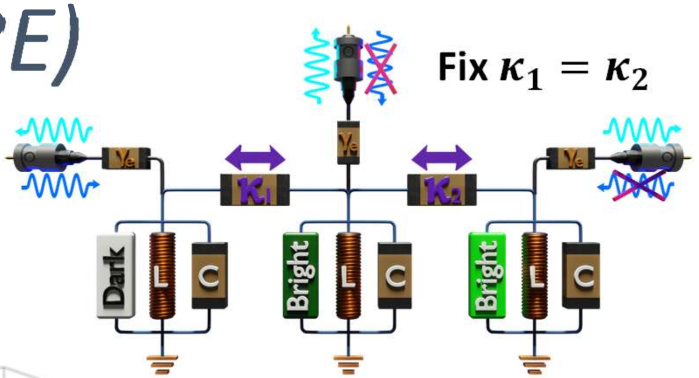


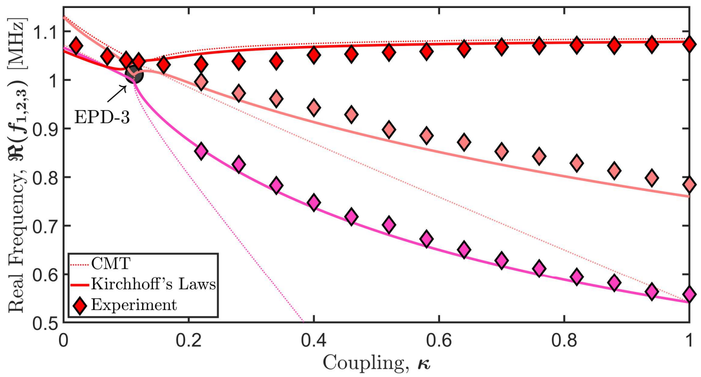


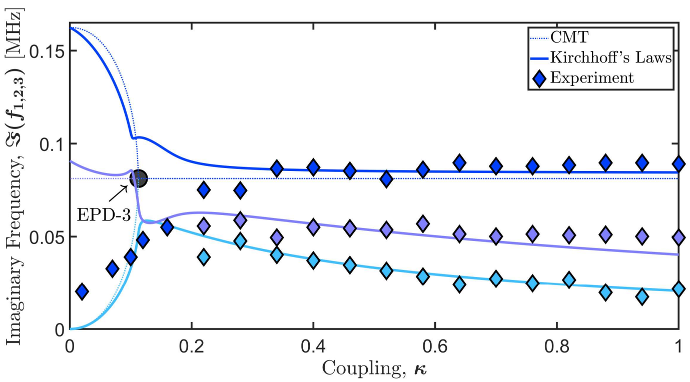


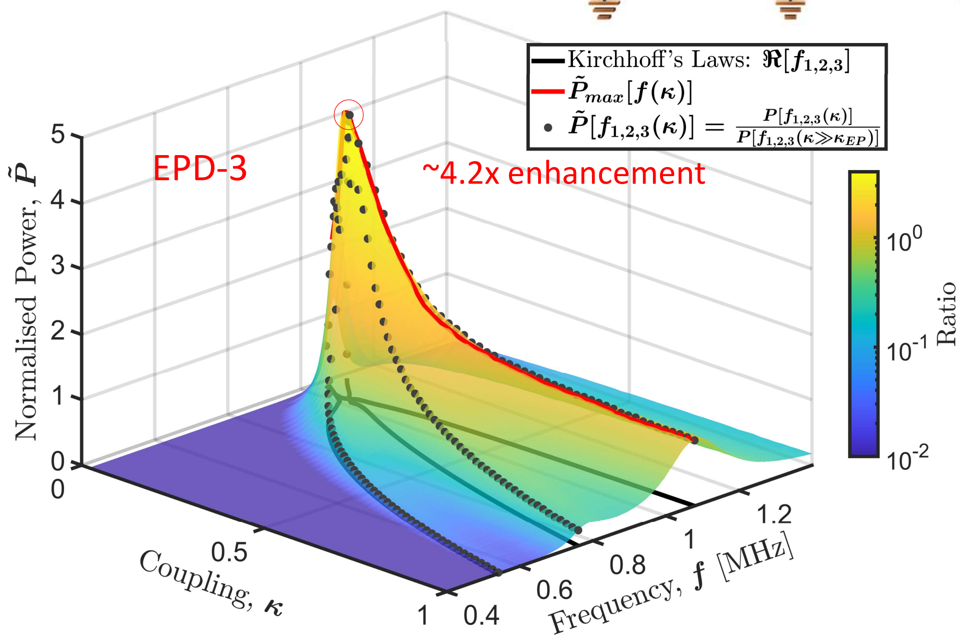


Trimer Experimental Results (SRE)









Conclusions

- EPDs enhance emission beyond the typical Purcell enhancement
- Both the order of the EPD and the fractional power expansion determine the non-Purcell enhancement
- We confirm our results via experiments with RLC electronic circuits