Wesleyan University
Department of Physics
Wave Transport in Complex Systems Lab

Thermalization in Complex Multimoded Optical Fibers

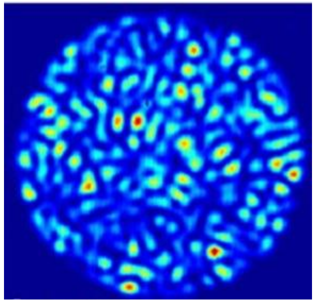
Emily Kabat

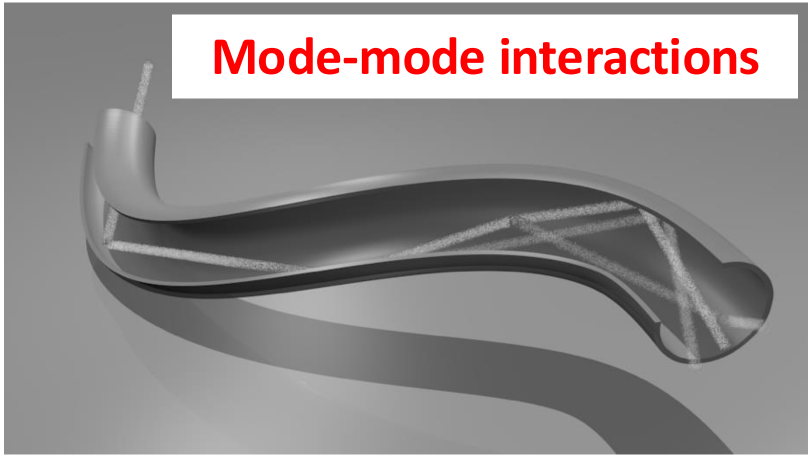
A. Ramos, L. Fernández-Alcázar, T. Kottos

E. Kabat *et. al., Nonlinear Defect Theory of Thermal Relaxation in Complex Multimoded Systems,* Phys. Rev. Research **6,** 033114
E. Kabat *et. al., Designing nonlinear beam relaxation rates via a Fluctuation-Dissipation relation,* in preparation.

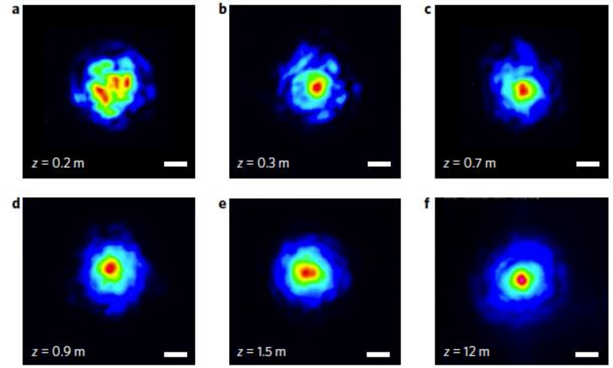
Beam Self-Cleaning in Nonlinear Multimode Fibers

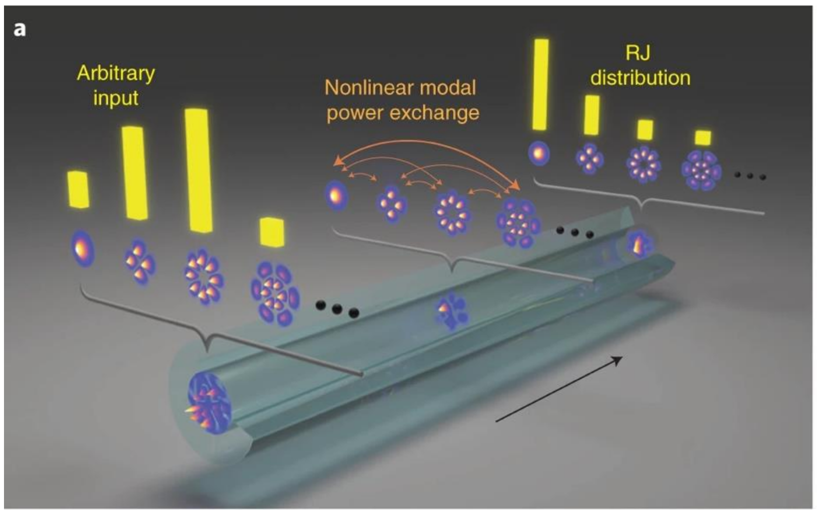
Input

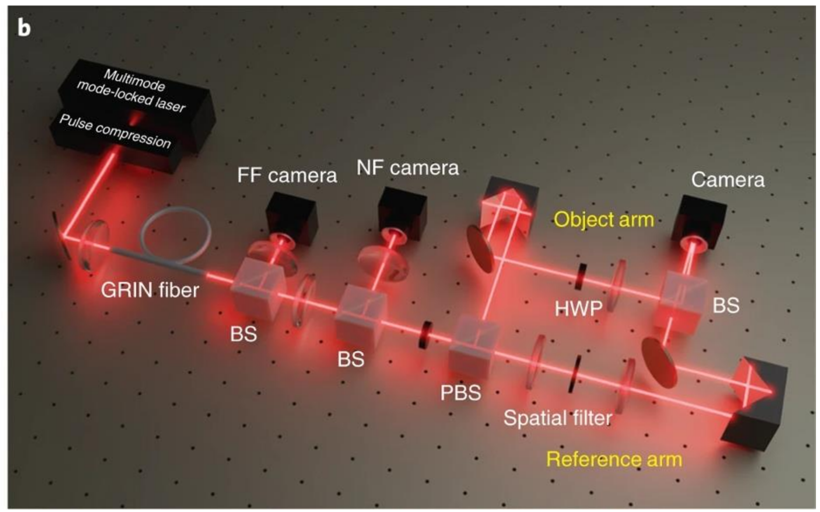




Output







L. Wright, D. Christodoulides, F. Wise, Opt. Lett. (2016)
K Krupa et al, Nature Photonics (2017)
H. Pourbeyran et. al., Nature Physics (2022)

Thermodynamic Description of MMFs

Two constants of motion:

Hamiltonian
(Energy)

$$\mathrm{H}\{\psi_l(z)\} = -\sum_{l,j} J_{l,j}\psi_l^*\psi_j + \frac{1}{2}\chi\sum_l \psi_l^4$$

$$\approx \sum_{\alpha=1}^{N} \epsilon_\alpha |C_\alpha|^2 = E$$

Norm/Power
(Number of Particles)

$$\mathcal{N}\{\psi_l(z)\} = \sum_{l=1}^{N} |\psi_l|^2$$

$$= \sum_{\alpha=1}^{N} |C_\alpha|^2 = A$$

Applying Grand Canonical:

$$\mathcal{Z} = \int \left(\prod_{l=1}^{N} d\psi_l^* d\psi_l \right) e^{-\beta[\mathcal{H}\{\psi_l(z)\} + \mu\mathcal{N}\{\psi_l(z)\}]}$$

$$= \Pi_k \left(\frac{\pi}{\beta(\epsilon_k - \mu)}\right)$$

Rayleigh-Jeans:

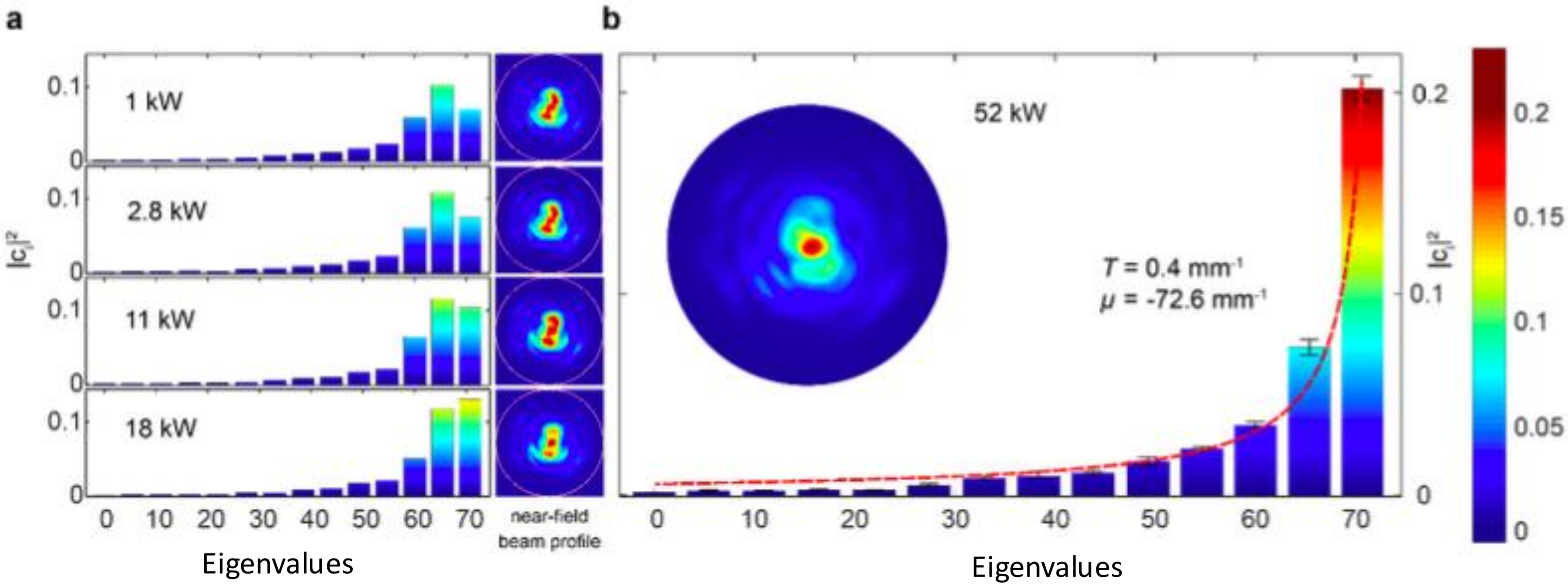
$$< |C_k|^2 > = \frac{1}{\beta(\epsilon_k - \mu)}$$

Mode Power → $|C_k|^2$

Eigenvalue → ϵ_k

Konstantinos, Wu, Jung, Christodoulides, Opt. Lett. 45, 1651-1654 (2020)
A. Ramos, L. Fernandez, T. Kottos, B. Shapiro, Phys. Rev. X 10, 031024 (2020)

Thermodynamic Description of MMFs



Thermodynamic predictions match experiment!

Rayleigh-Jeans:

$$< |C_k|^2 > = \frac{1}{\beta(\epsilon_k - \mu)}$$

Mode Power

Eigenvalue

Konstantinos, Wu, Jung, Christodoulides, Opt. Lett. 45, 1651-1654 (2020)
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Relaxation Rates Γ

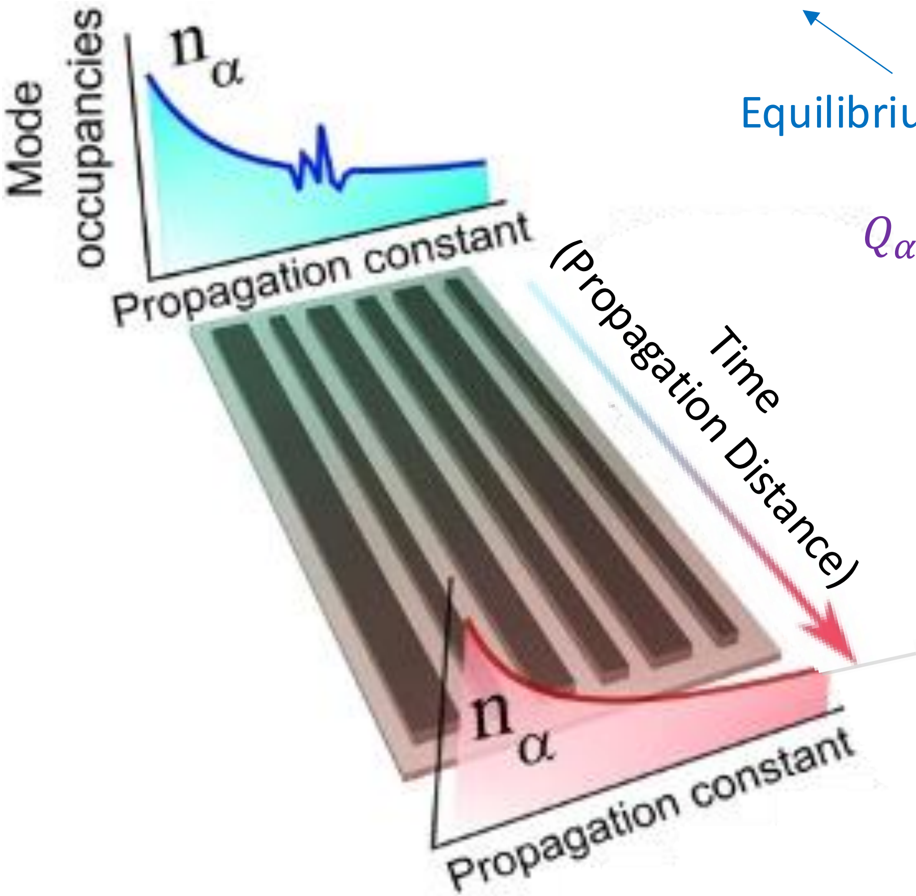
Nonlinear Strength

Four-Wave Overlap

The α^{th} mode relaxes like $\Gamma_\alpha = \frac{4\pi\chi^2}{n_\alpha} \sum'_{\beta\gamma\delta} \left| Q_{\alpha\beta\gamma\delta} \right|^2 n_\beta n_\gamma n_\delta \delta(\epsilon_\alpha + \epsilon_\beta - \epsilon_\gamma - \epsilon_\delta)$

Equilibrium Mode Powers

Eigenvalue Distribution



$$Q_{\alpha\beta\gamma\delta} = \sum_{\substack{m \in \\ nonlinear \\ sites}} f_\alpha^*(m) f_\beta^*(m) f_\gamma(m) f_\delta(m)$$

Assumptions:

- Weak nonlinearity
- Random Phase and Amplitude
- Localization lengths > lattice spacing
- No condensation

Relaxation Rates Γ

Nonlinear Strength

Four-Wave Overlap

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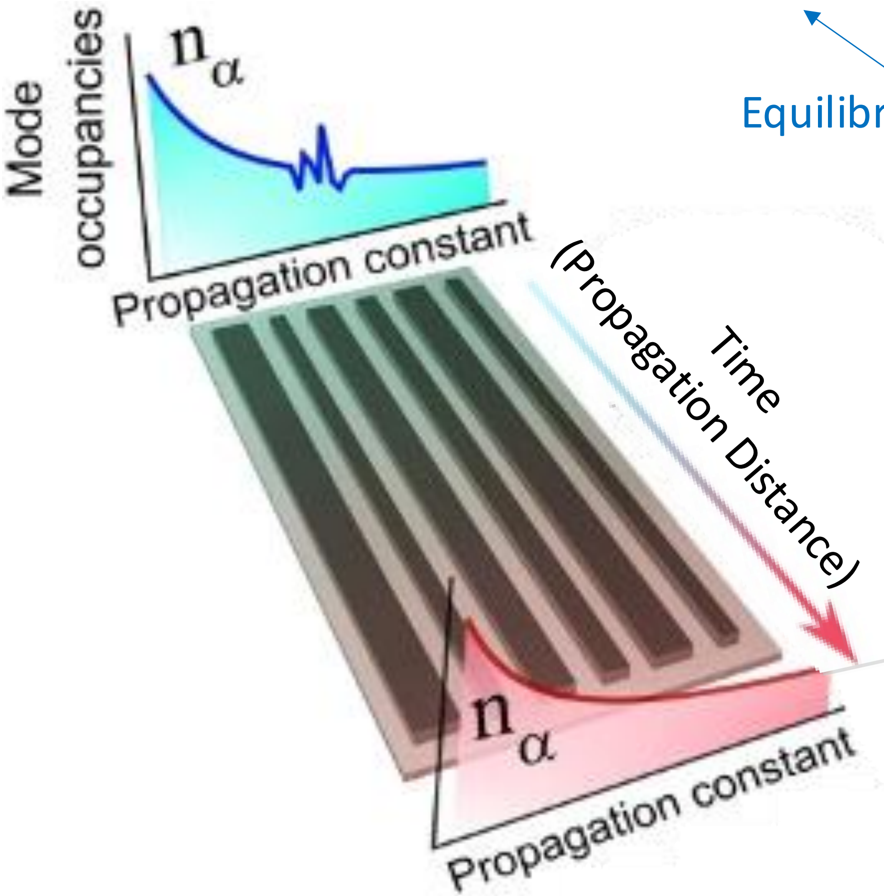
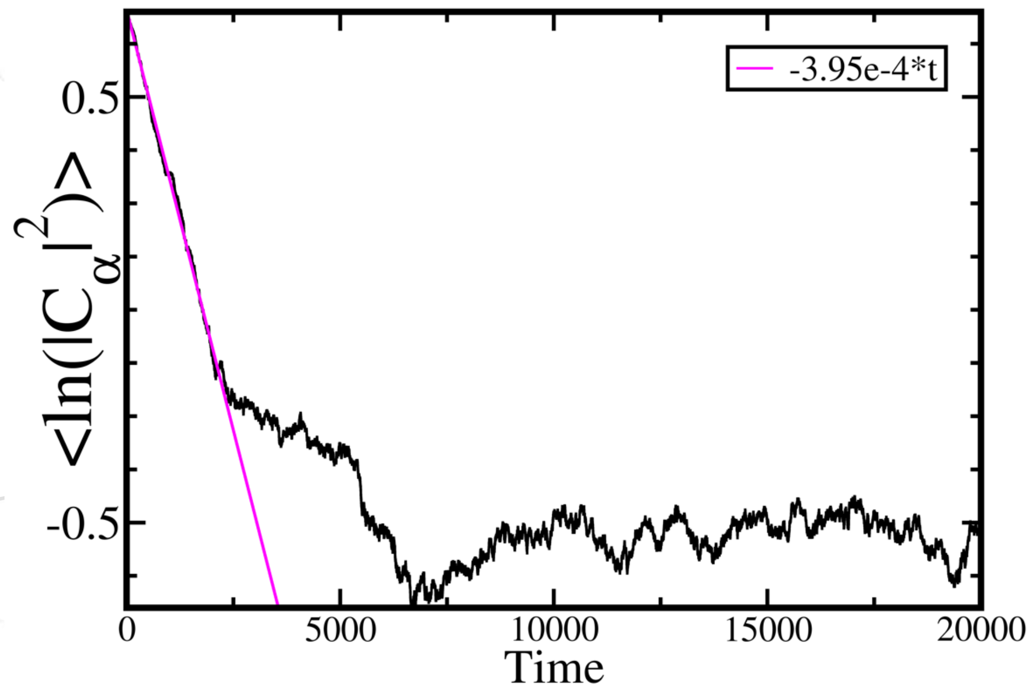




Relaxation Rates—Nonlinear Defects

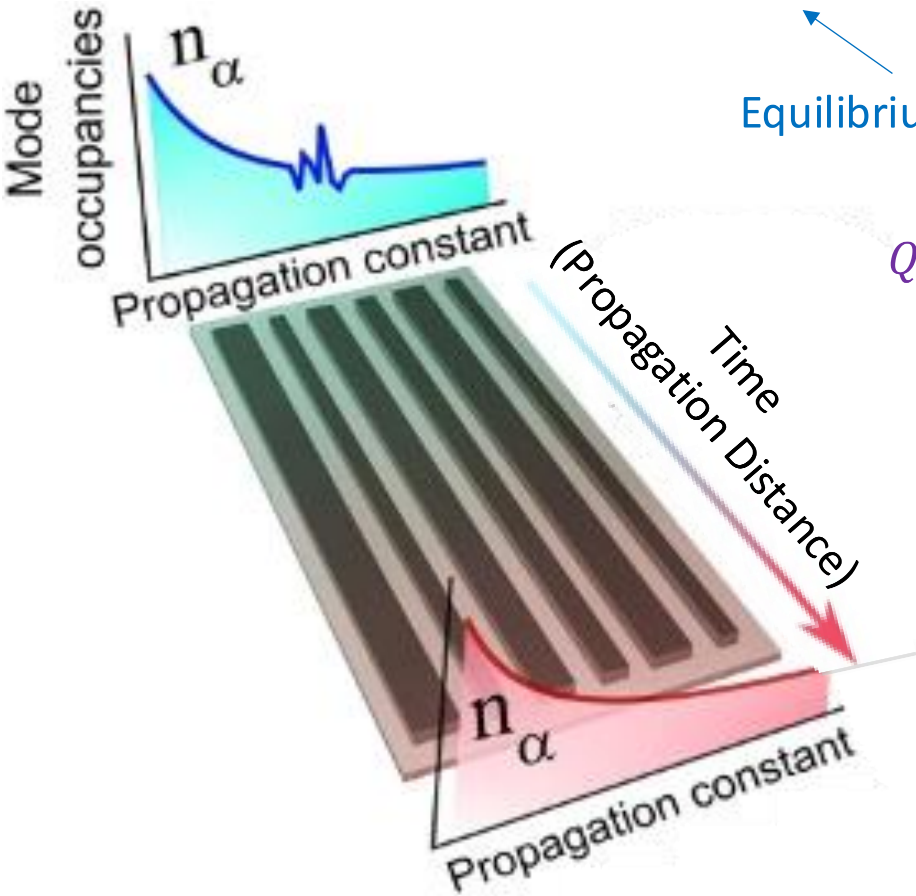
Nonlinear Strength

Four-Wave Overlap

The α^{th} mode relaxes like $\Gamma_\alpha = \frac{4\pi\chi^2}{n_\alpha} \sum'_{\beta\gamma\delta} \left|Q_{\alpha\beta\gamma\delta}\right|^2 n_\beta n_\gamma n_\delta \delta(\epsilon_\alpha + \epsilon_\beta - \epsilon_\gamma - \epsilon_\delta)$

Equilibrium Mode Powers

Eigenvalue Distribution



$$Q_{\alpha\beta\gamma\delta} = \sum_m f_\alpha^*(m) f_\beta^*(m) f_\gamma(m) f_\delta(m) \boldsymbol{\delta(m - m_0)}$$

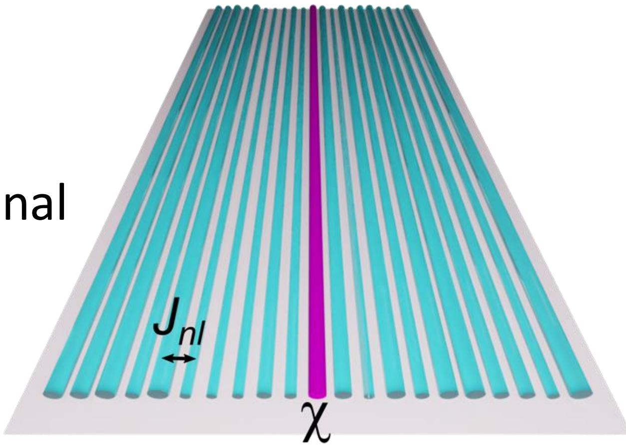
For systems with a nonlinear defect,

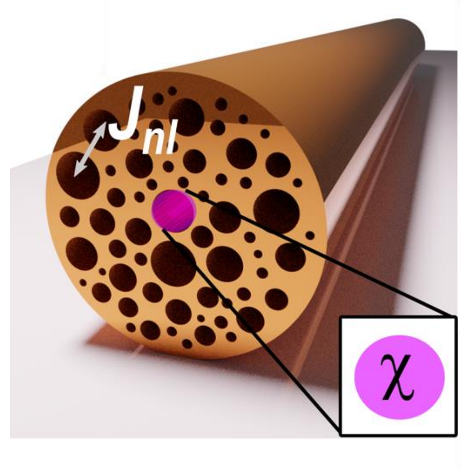
Assume $\Gamma_\alpha \propto \chi^2 F_\alpha(T, \mu) \times |f_\alpha(m)|^2$

Analysis of relaxation rates collapses to analysis of mode amplitude distributions

Relaxation Rates—Nonlinear Defects

One-Dimensional

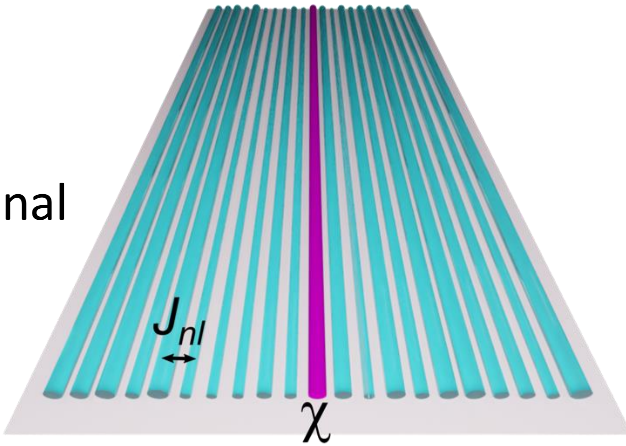


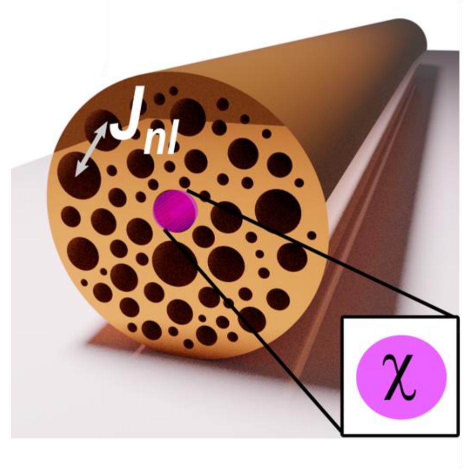


Quasi One-Dimensional

Relaxation Rates—Nonlinear Defects

One-Dimensional





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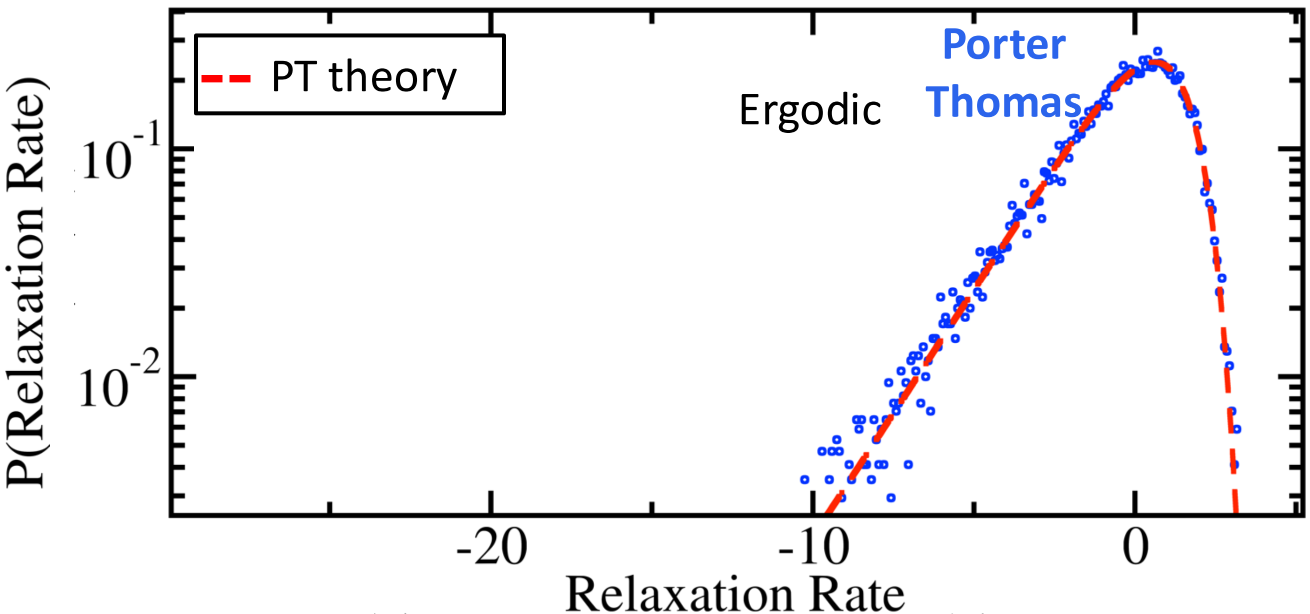
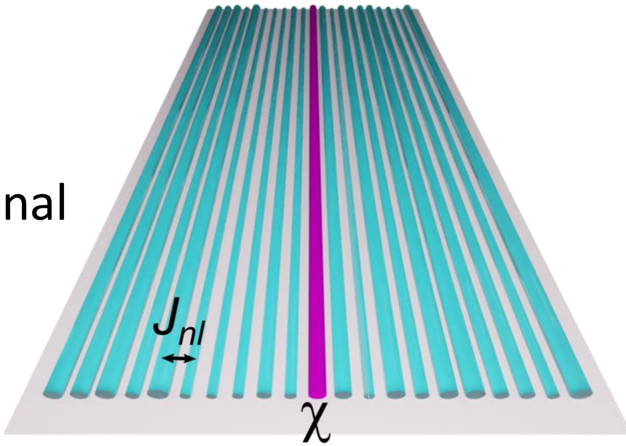
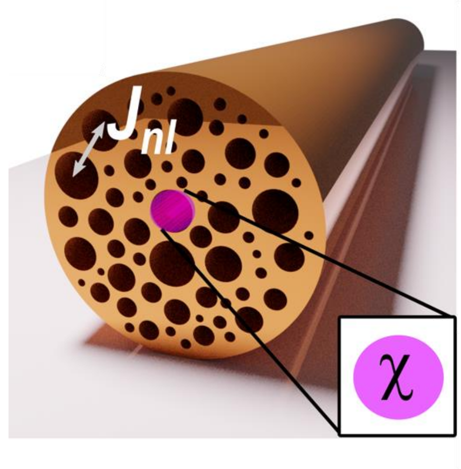


Relaxation Rates—Nonlinear Defects

One-Dimensional





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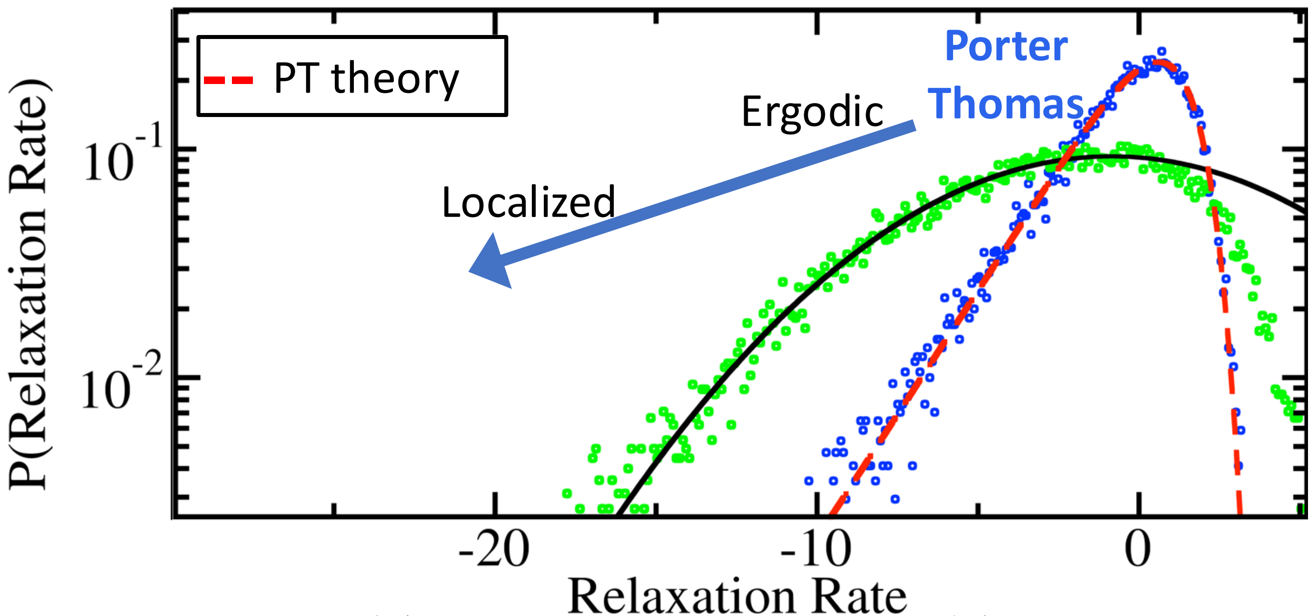
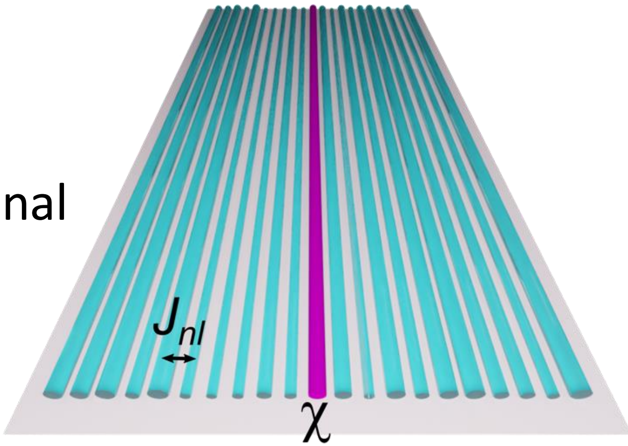
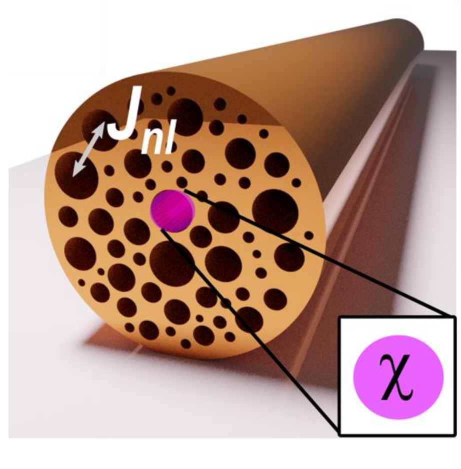


Relaxation Rates—Nonlinear Defects

One-Dimensional





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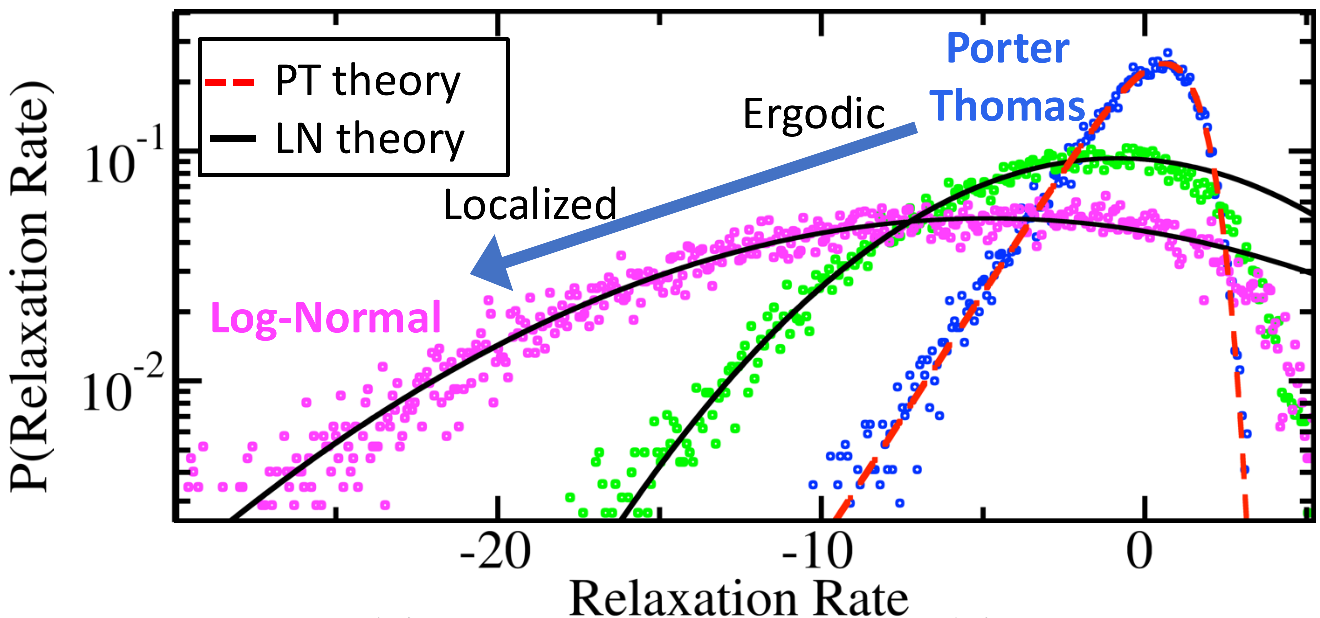


Relaxation Rates—Correlation-Based Analysis

$$\Gamma = \langle \Gamma_\alpha \rangle_\alpha \approx \frac{4\pi\chi^2}{N}\left(\frac{T}{\mu}\right)^2 \sum_{\alpha\beta\gamma\delta}' \left| \sum_m f_\alpha^*(m) f_\beta^*(m) f_\gamma(m) f_\delta(m) \right|^2 \delta(\epsilon_\alpha + \epsilon_\beta - \epsilon_\gamma - \epsilon_\delta)$$

Nonlinear Strength (χ^2)

Equilibrium Mode Powers (T/μ)

Four-Wave Overlap ($\left|\sum_m f_\alpha^*(m) f_\beta^*(m) f_\gamma(m) f_\delta(m)\right|^2$)

Eigenvalue Distribution ($\delta(\epsilon_\alpha + \epsilon_\beta - \epsilon_\gamma - \epsilon_\delta)$)

Correlation $\mathcal{C}_m(\omega) = \sum_{\alpha,\gamma} \left\langle |f_\alpha(m)|^2 \left|f_\gamma(m)\right|^2 \delta\left(\varepsilon_\alpha - \varepsilon_\gamma - \omega\right) \right\rangle$

$$\Gamma/\chi^2 \approx 4\pi\left(\frac{T}{\mu}\right)^2 \times \frac{1}{N}\sum_m \int \mathrm{d}\omega\, \mathcal{C}_m^2(\omega)$$

Fluctuation-Dissipation Relation!

The relaxation rates again simplify to mode statistics!

Relaxation Rates—Correlation-Based Analysis

Network Topology

(disorder, connectivity, etc.)

↓

Mode-Mode Correlations

↓

Relaxation Rates

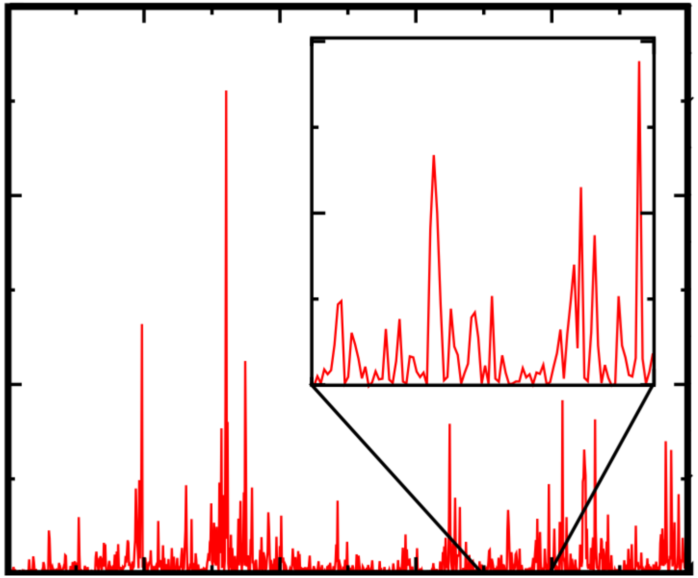
Multifractal Modes

Greece's coastline has fractal dimension ≈ 1.25

The shorter you make your ruler, the more distance you measure!

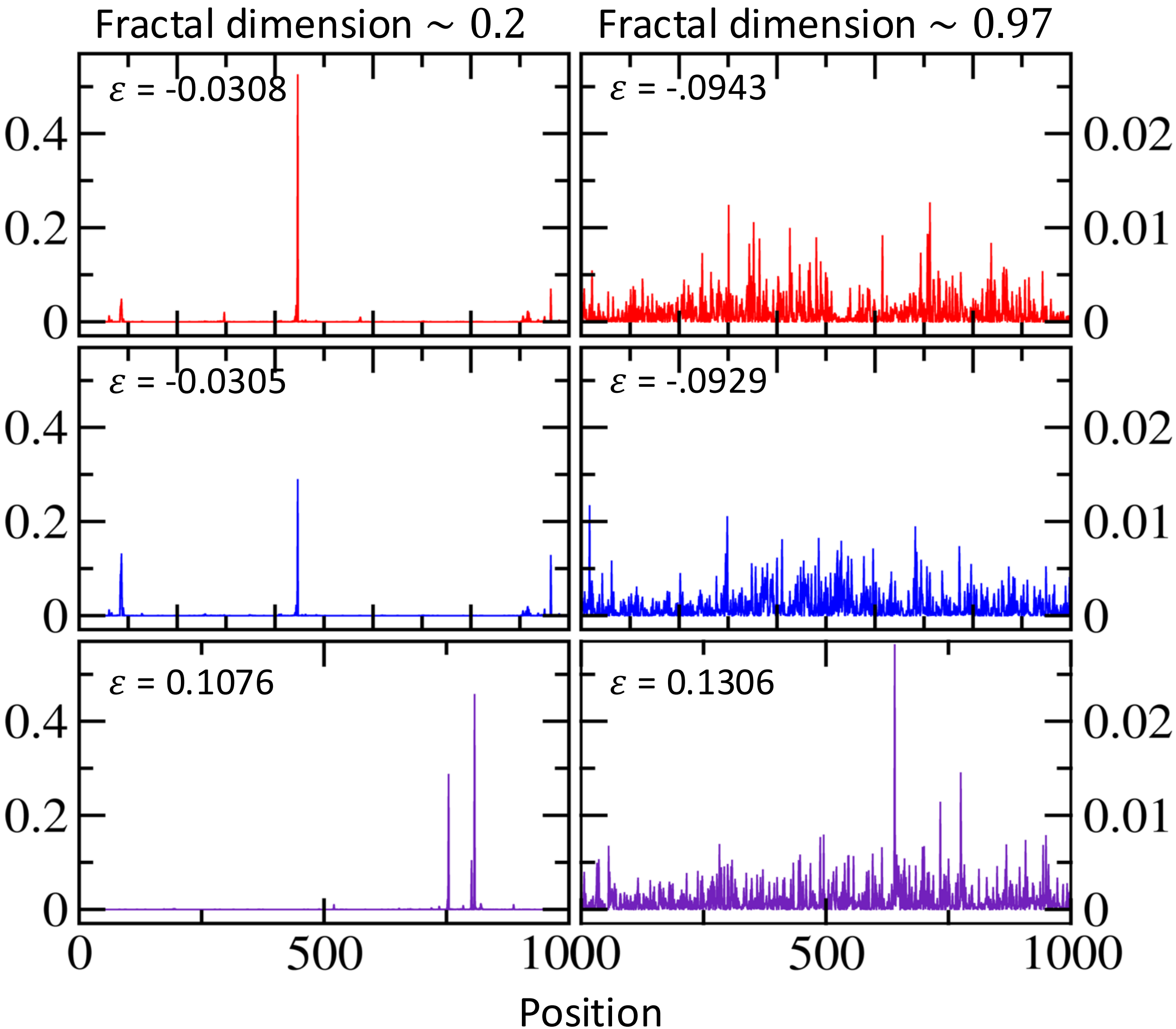
Multifractal modes have…

- Self-similarity
- Anomalous scaling with system size



Space occupied by mode $\sim N^{d_2}$, $d_2 \notin \mathbb{Z}$

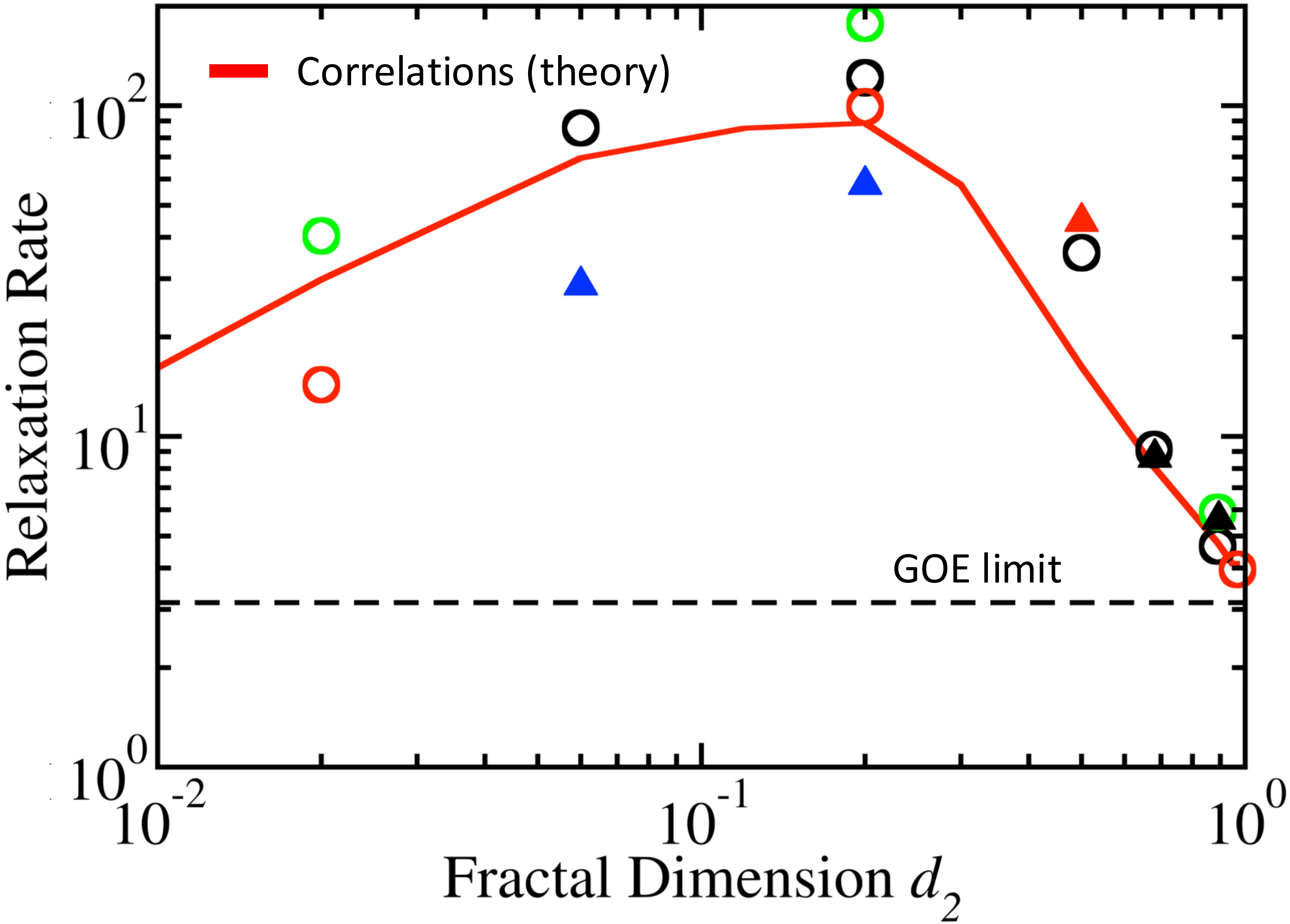
Correlations of Multifractal Modes



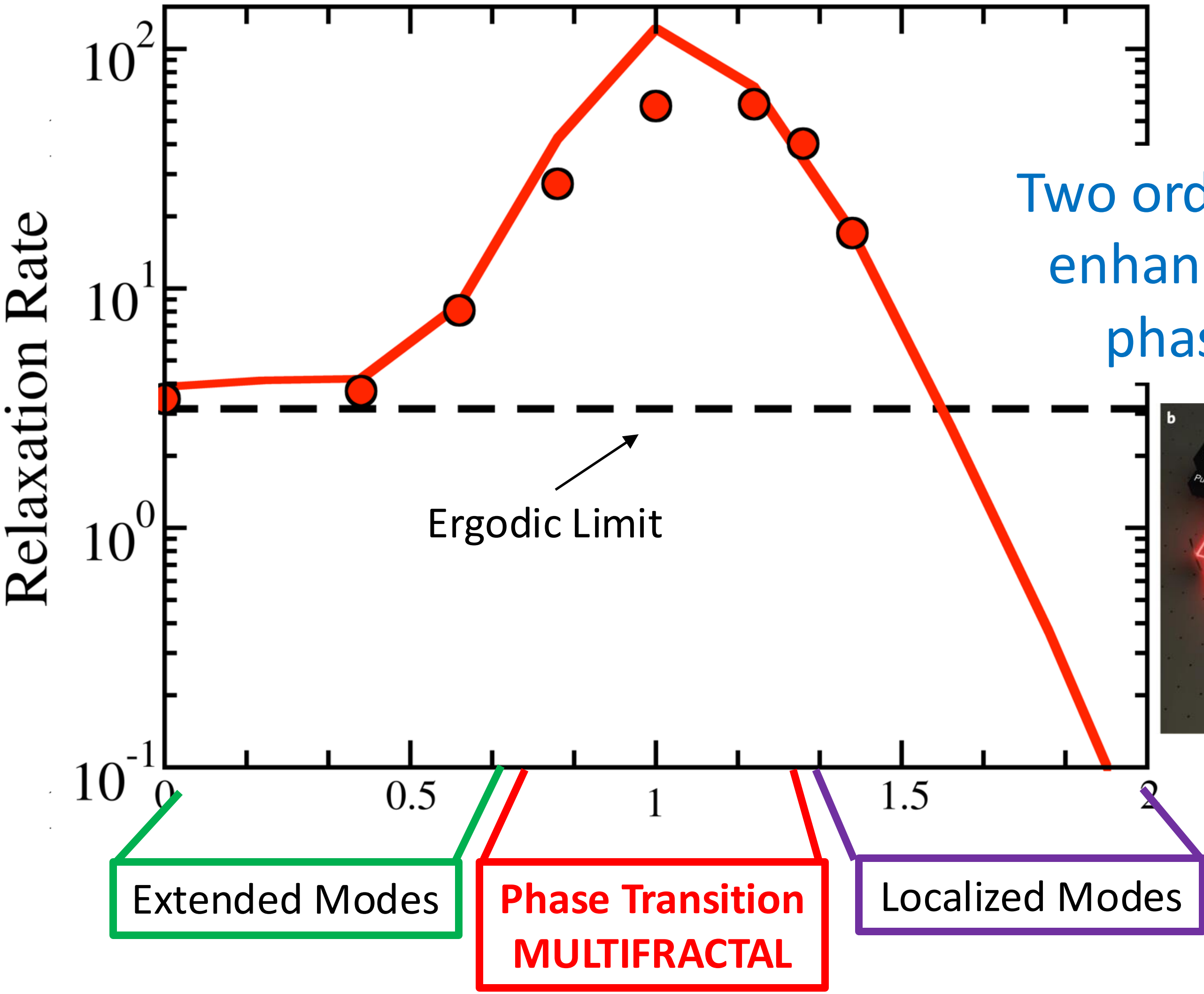
Fractal modes have anomalous correlations

Acceleration of Thermal Relaxation

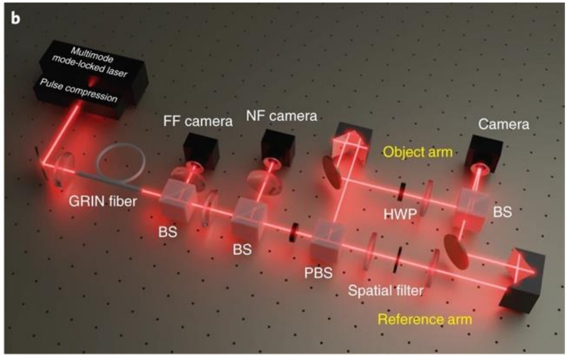
$$\Gamma = 4\pi \left(\frac{T}{\mu}\right)^2 \overline{\mathcal{C}^2}$$



Acceleration of Thermal Relaxation

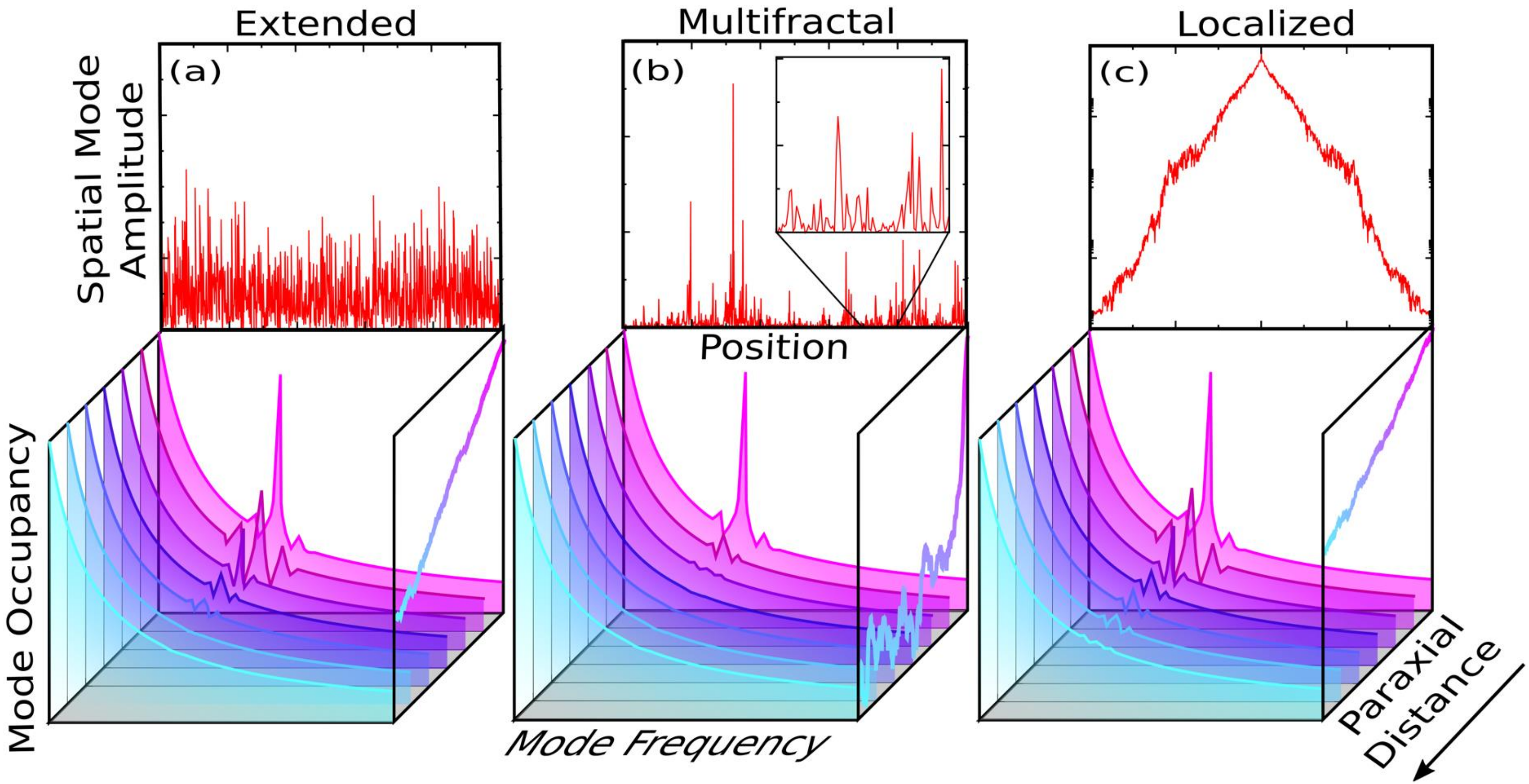


Two order of magnitude enhancement around phase transition!



Acceleration of Thermal Relaxation

$$\mathcal{C}_m(\omega) = \sum_{\alpha,\gamma} \left\langle |f_\alpha(m)|^2 \left|f_\gamma(m)\right|^2 \delta\left(\varepsilon_\alpha - \varepsilon_\gamma - \omega\right) \right\rangle$$



Thank you!