Broad Band Unidirectional Invisibility using $\mathcal{PT}$-Symmetry

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Abstract: We show that parity-time symmetric Bragg grating structures, at the spontaneous $\mathcal{PT}$-symmetry point, can act as unidirectional transparent media where the reflection from one side is suppressed while it is enhanced from the other.

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One of the key challenges encountered in many integrated optics arrangements is the presence of substantial level of absorption, that typically degrades the efficiency of optical devices. As a result, considerable research effort is invested in eliminating and mitigating these undesirable absorption mechanisms. An alternative proposal, introduced very recently [1], adopts instead a different viewpoint: it was proposed to manipulate loss via a judicious design that involves the combination of symmetrically balanced amplification and absorption regions. In addition, it has been demonstrated [2,3] that one can synthesize new classes of optical materials with altogether different characteristics. Among the numerous intriguing features that these systems can display, are: power oscillations [1], absorption enhanced transmittivity [3], double refraction and non-reciprocity of light propagation [1]. At the same time, such designs based on the manipulation of gain and loss can allow for a precise tailoring of light flow, the creation of on-chip integrated optical isolators [4], and of cloaking devices. The suggested synthetic materials can be realized in optics, by creating a medium with a complex refractive index that satisfies the condition $n^*(x)=n(-x)$. In other words, the refractive index satisfies a combined Parity ($\mathcal{P}$) and Time ($\mathcal{T}$) symmetry which essentially implies that the real part of the refractive index is an even function of position, whereas the imaginary part is odd. The above constraint indicates that creation and absorption of photons occur in a balanced manner, so that the net loss or gain is zero. Reference [2] reports the first successful realization of such $\mathcal{PT}$-synthetic materials.

To date, most of the optical realizations of $\mathcal{PT}$– synthetic media have relied on the paraxial approximation which maps the scalar wave equation to the Schroedinger equation, with the axial wavevector playing the role of energy. This formal analogy, allow us to investigate experimentally fundamental $\mathcal{PT}$-concepts that impact several other areas, ranging from quantum field theory [5] and mathematical physics, to solid state [6] and atomic physics. Among the various themes that fascinate researchers, is the existence of a spontaneous $\mathcal{PT}$ symmetry braking where the eigenvalues of the effective non-Hermitian Hamiltonian describing the dynamics of $\mathcal{PT}$ systems turn from real to complex and the bi-orthogonal eigenmodes coalesce (at the exceptional point), the signatures of $\mathcal{PT}$ symmetry in scattering, etc.

In this paper we study light propagation through $\mathcal{PT}$– synthetic media in a new setting, namely light scattering from $\mathcal{PT}$-fiber Bragg gratings ($\mathcal{PT}$-FBG). Passive (no gain/loss) FBG act as reflectors around the Bragg wavelength while transmitting all others. Here, we consider the consequences of $\mathcal{PT}$ – i.e. the nature of the medium in the scattering regime. Specifically, we show that at the spontaneous $\mathcal{PT}$ – symmetry breaking point, the system is fully transparent (i.e. has zero reflection) and thus “invisible”, if the light is incident from one side of the structure while from the other side visibility(i.e. reflectivity) is enhanced. The phenomenon is robust for a broad band wavelengths near the Bragg point, and for gain-loss arrangements relatively close to the spontaneous $\mathcal{PT}$ – symmetry breaking.
point. Even more surprisingly, is the fact that it survives even in the presence of Kerr non-linearity.

We begin by considering a periodic system having a uniform refractive index \( n_0 \) and an embedded complex grating domain \(|z|<L/2\) where the refractive index is \( n(z)=n_0+n_1 \cos(2\beta z)+i n_2 \sin(2\beta z)\). In this configuration the grating period is \( \pi/\beta \) and \( n_1, n_2 \ll n_0 \) is the amplitude of the (complex) grating where the imaginary part of \( n(z) \) describes the local gain/loss of the medium. This grating exhibits a spontaneous \( PT \) symmetry breaking for \( n_1=n_2 \). For \( n_2=0 \) the periodic modulation of refractive index leads to Bragg reflection at frequencies close to the Bragg frequency, \( \omega_0=\pm \beta/\hbar n_0 \) where \( c \) is the speed of light in vacuum. The steady state scattering solution for the electric field \( E(z) \) with frequency \( \omega \) obeys the Helmholtz equation

\[
\frac{d^2 E}{dz^2}+(\omega/c^2)n^2(z)E=0
\]  

Outside the grating domain Eq.1 admits a solution \( E_0(z)=A_+e^{ikz}+B_+e^{-ikz} \) for \( z<-L/2 \) and \( E_0^*(z)=A_+e^{ikz}+B_+e^{-ikz} \) for \( z>L/2 \) where \( k=n_0\omega/c \). The corresponding transmission and reflection coefficients, for left (right) incident waves corresponding to \( B_+=0(A_+=0) \) are \( T_r=|A_+/A_0|^2 \) \( (T_t=|B_-/B_0|^2) \), \( R_t=|B_+/A_0|^2 \) \( (R_r=|A_+/B_0|^2) \). By decomposing the electric field inside the scattering domain \(|z|<-L/2\), in terms of forward \( E_f(z) \) and backward \( E_b(z) \) traveling envelopes as \( E_f(z)e^{ikz}+E_b(z)e^{-ikz} \) and by employing coupled mode theory, we find exact expressions for the transmission and reflection coefficients near the Bragg frequency. Specifically at the exceptional point \( n_1=n_2 \) we find that \( T_f=T_t=1 \) and \( R_r=0 \) (i.e. perfect transmission) while \( R_f \propto L^2 \). The latter algebraic amplification of \( R_f \) is a characteristic of \( PT \) media at the exceptional point [7]. Detailed simulations with Eq.1 (see Figure 2a), allow us to conclude that the Unidirectional Invisibility (UI) survives even for a broad range of frequencies in the vicinity of the Bragg frequency and is robust against small deviations from the exceptional point. Even more surprising is the fact that UI can be displayed also in the presence of Kerr nonlinearity indicating that the phenomenon is not related to interference effects but rather it is the outcome of the interplay of Bragg scattering and \( PT \) - symmetry (see Figure 2b).

\[
\begin{align*}
\begin{array}{c}
\text{(a)} \\
\begin{array}{c}
\text{n} \_1=1, \text{n} \_0=0.1, \text{L}=10, \beta=5
\end{array}
\end{array}
\end{align*}
\]

\[
\begin{align*}
\begin{array}{c}
\text{(b)} \\
\begin{array}{c}
\text{L}=7, \text{n} \_0=1, \text{n} \_1=0.5, \delta=0 \\
\text{L}=7, \text{n} \_0=1, \text{n} \_1=0.5, \delta=-5
\end{array}
\end{array}
\end{align*}
\]

Fig. 2. (a)Unidirectional Transparency (UI), linear case. (b) Transmission and reflection in the nonlinear regime vs. right(negative numbers) and left (positive numbers) input intensity. The green line correspond to UI.

In conclusion, we have shown that the interplay of Bragg scattering and \( PT \) - symmetry allows for unidirectional transparency/invisibility which can be observed in a broad frequency range around the Bragg frequency. This happens to be robust against small perturbations around the exceptional point. Unidirectional invisibility is present even in the case of Kerr nonlinearities. It will be interesting to find out if the phenomenon presented here still persists in higher dimensions.

References: